B. CORRELATIONS FOR AVERAGE GAS HOLD-UP

The average gas hold-up values measured for the different waxes under the various conditions were divided into the following four groups:

Group 1: Data from experiments conducted in the "foamy" regime using orifice type distributors (in the 0.051 m ID column), and perforated plate or perforated pipe type of distributors (in the 0.229 m ID column).

Group 2: Data from experiments conducted in the "slug flow" regime using orifice type distributors (in the 0.051 m ID column), and in the "churn-turbulent" regime using perforated plate or perforated pipe type of distributors (in the 0.229 m ID column).

Group 3: Data from experiments conducted in the "foamy" regime using the 40 μm SMP distributor (in the 0.051 m ID column).

Group 4: Data from experiments conducted in the "slug flow" regime using the 40 μm distributor (in the 0.051 m ID column).

Various correlations were used with each of these groups and the goodness of fit was determined. The average gas hold-up values in all correlations presented here are expressed on a percentage basis (i.e. percent of the dispersion occupied by the gas phase).

The major findings from this section are:

- Hold-up values in the "foamy" regime with orifice and perforated plate distributors show a dependency on column diameter and distributor type, and can be predicted by:

$$\epsilon_g\ (\%) = 12(Bo)^{-0.19}(We)^{0.13}(Ga)^{0.11}(Fr)^{0.19} \qquad 0.01 \le u_g \le 0.07\ m/s \qquad \text{(VI-14)}$$

- Hold-up values in the "slug flow" regime (in the 0.051 m ID column) and in the "churn-turbulent" regime (in the 0.229 m ID column) are independent of column diameter and distributor type and can be predicted by:

$$\epsilon_g\ (\%) = 25(Bo)^{0.15}(Fr)^{0.60} \qquad 0.01 \le u_g \le 0.15\ m/s \qquad (VI\text{-}16)$$

- Hold-up values in the "foamy" regime with the SMP distributor show a weak dependence on superficial gas velocities greater than 0.02 m/s and can be predicted by:

$$\epsilon_g\ (\%) = \frac{10560u_g}{1+130u_g} \qquad 0.01 \le u_g \le 0.12\ m/s \qquad (VI\text{-}17)$$

- Hold-up values in the "slug flow" regime with the SMP distributor show an increase with an increase in the superficial gas velocity and can be predicted by:

$$\epsilon_g\ (\%) = 98u_g^{0.61} \qquad 0.01 \le u_g \le 0.12\ m/s \qquad (VI\text{-}18)$$

B.1. Correlations Based on Dimensional Analysis

The factors affecting the gas hold-up are the physical properties of the liquid medium, ρ_ℓ (density), μ_ℓ (viscosity) and σ (surface tension); gas density, ρ_g; column diameter, d_c, superficial gas velocity, u_g; jet velocity, u_j; orifice diameter, d_o; and gravity, g. By dimensional analysis, the following functional form for the average gas hold-up is obtained:

$$\epsilon_g = fn\ [Bo,\ We,\ Ga,\ Fr] \qquad (VI\text{-}1)$$

where

$$Bo = \frac{d_c^2 \rho_\ell g}{\sigma} \qquad \text{Bond number} \qquad (VI\text{-}2)$$

$$We = \frac{d_o \rho_g u_j^2}{\sigma} \qquad \text{Weber number} \qquad (VI\text{-}3)$$

$$Ga = \frac{d_c^3 g \rho_\ell^2}{\mu_\ell^2} \qquad \text{Galileo number} \qquad (VI\text{-}4)$$

$$Fr = \frac{u_g}{\sqrt{d_c g}} \quad \text{Froude number} \tag{VI-5}$$

In Equation (VI-1), the Weber number (We) is based on the orifice diameter and the jet velocity through the orifice. Therefore, for correlations involving hold-up for SMP distributors, this quantity is not applicable and Equation (VI-1) reduces to:

$$\epsilon_g = fn\ [Bo,\ Ga,\ Fr] \tag{VI-6}$$

Hold-up correlations were also developed using a modified form of Equation (VI-1). Equation (VI-7) was used for orifice and perforated plate type of distributors, and Equation (VI-8) was used for the SMP distributor.

$$\frac{\epsilon_g}{100-\epsilon_g} = fn\ [Bo,\ We,\ Ga,\ Fr] \tag{VI-7}$$

$$\frac{\epsilon_g}{100-\epsilon_g} = fn\ [Bo,\ Ga,\ Fr] \tag{VI-8}$$

The range of values for the various parameters and dimensionless groups used in these correlations are summarized in Table VI-2.

B.2. Empirical Hold-up Correlations

Two empirical correlations were selected to fit hold-up values obtained during the present study. These correlations relate the average gas hold-up to the superficial gas velocity.

$$\epsilon_g = k_1 u_g^{k_2} \tag{VI-9}$$

$$\epsilon_g = \frac{k_1 u_g}{1 + k_2 u_g} \tag{VI-10}$$

The first of these correlations (Equation (VI-9)) was used to describe hold-up data by various researchers (e.g. Deckwer et al., 1980).

Table VI-2. Range of values for various parameters used in the correlations

PARAMETER	RANGE
liquid density (ρ_ℓ)	655 - 730 kg/m^3
liquid viscosity (μ_ℓ)	1.9 - 6.4 mPa.s
surface tension (σ)	0.020 - 0.028 N/m
gas density (ρ_g)	0.660 - 0.843 kg/m^3
column diameter (d_c)	0.051, 0.229 m
superficial gas velocity (u_g)	0.01 - 0.17 m/s
jet velocity (u_j)	1.63 - 331 m/s
orifice or perforated plate pore size (d_o)	1 - 4 mm
Bond number (Bo)	703 - 16738
Weber number (We)	0.058 - 3282
Galileo number (Ga)	3.82×10^7 - 1.28×10^{10}
Froude number (Fr)	0.0067 - 0.2403

B.3. Correlations from Literature

A large number of correlations for gas hold-up have been proposed in the literature based on measurements made with various gas-liquid systems. The correlations presented by Akita and Yoshida (1973), Bach and Pilhofer (1978), Mersmann (1978) and Hikita et al. (1980) are based on a large number of experimental data and are often used to predict the average gas hold-up. These correlations were used to predict hold-ups for the waxes used in the present study and it was found that Bach and Pilhofer's correlation provides a very good fit for data obtained in the "slug flow" regime (in the 0.051 m ID column) and the "churn-turbulent" regime (in the 0.229 m ID column). This correlation is based on measurements made using pure organic liquids as the liquid media. Figure VI-1 compares the values predicted by Bach and Pilhofer's correlation to those obtained in the present study at 265°C. The physical properties of FT-300 wax at 265°C were used in the correlation. Also included in this figure are results from experiments conducted by Calderbank et al. (1963) with Krupp wax using a ball and cone type of distributor, and from experiments conducted by Kuo et al. (1985) with FT-200 wax using a 2 mm orifice plate distributor. The agreement between the experimental and predicted values is very good. Therefore, this correlation was selected and the constants reevaluated for our data:

$$\frac{\epsilon_g}{100-\epsilon_g} = k_1 \left[\frac{u_g^3 \rho_\ell^2}{\mu_\ell g(\rho_\ell - \rho_g)} \right]^{k_2} \qquad \text{(VI-11)}$$

Akita and Yoshida's (1973) correlation is based on measurements made using water, methanol, and glycol solution as the liquid media. The hold-up values predicted by their correlation were significantly lower than those

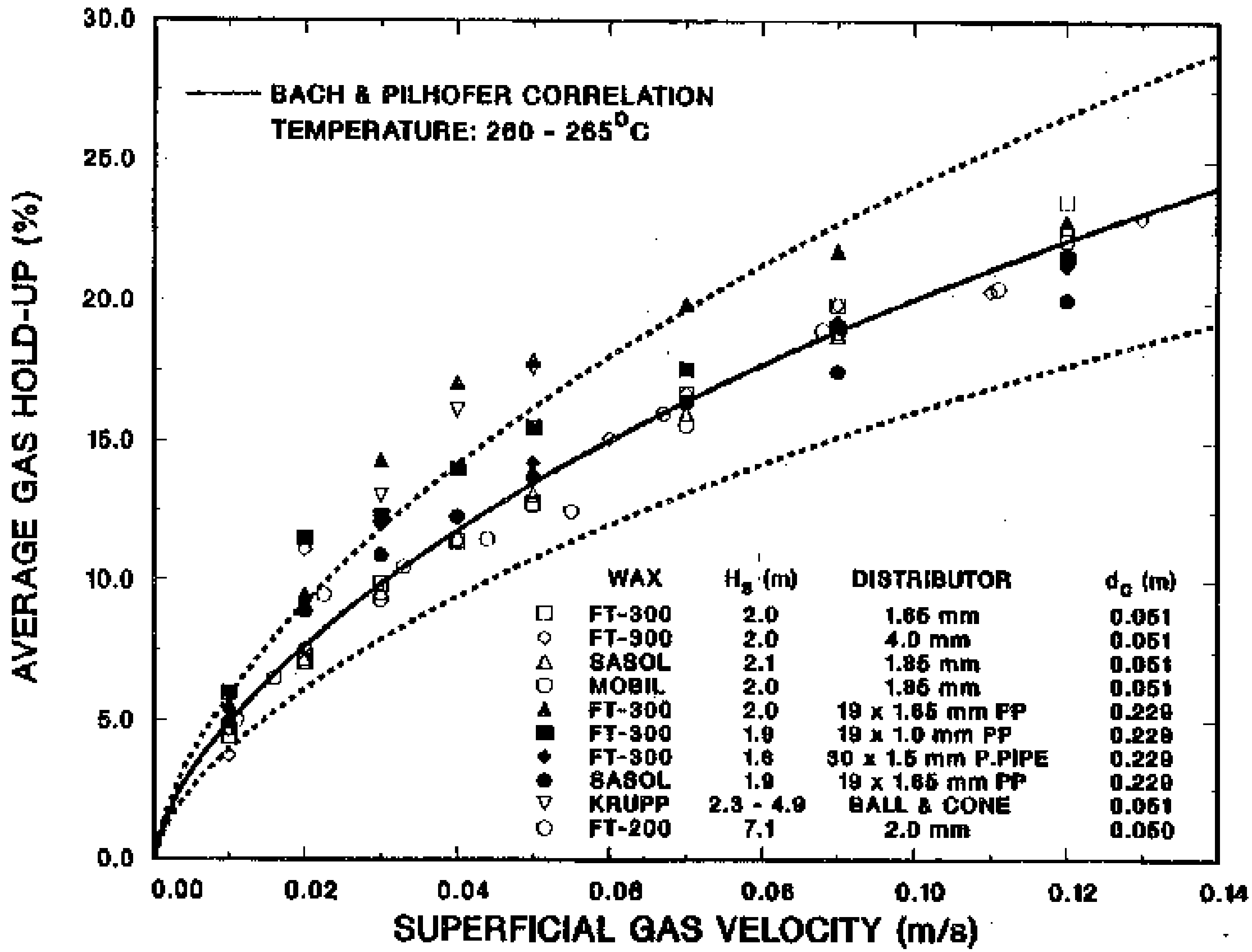

Figure VI-1. Comparison between experimental gas hold-ups and predictions from the Bach and Pilhofer correlation (solid line - Bach and Pilhofer correlation; dashed lines - ±20% relative deviation from the correlation; △ - Calderbank et al., 1963; ○ - Kuo et al., 1985; remaining data are from present study) obtained in the present study, however, the dimensionless groups involved with this correlation are similar to those presented earlier (Equation (VI-1)). Therefore, the constants for their correlations were reevaluated using data from the present study. The functional form of this correlation is:

$$\frac{\epsilon_g}{(100-\epsilon_g)^4} = \text{fn}\ [\text{Bo, Ga, Fr}] \tag{VI-12}$$

B.4. <u>Tests for Goodness of Fit</u>

The constants for the different correlations were evaluated using non-linear regression (i.e. the NLIN procedure in the SAS package) of the hold-up data. In order to compare the various correlations two different criteria were used. The first of these is the Mean Square Error (MSE) and is given as:

$$\text{MSE} = \frac{\sum_{i=1}^{N} (\epsilon_g - \epsilon_{gp})^2}{N} \tag{VI-13}$$

where ϵ_g is the actual or measured hold-up, ϵ_{gp} is the hold-up value predicted by the correlation, and N is the number of data points used to obtain the correlation. The second criterion is the percent of data points that are within ±30% of the values predicted by the correlation. The correlation with the lowest MSE and the greatest percentage of data points within ±30% provides the best representation of the experimental data.

B.5. <u>Results from Statistical Analysis</u>

The six correlations presented earlier were used with each of the four data groups. Tables VI-3 to VI-5 summarize results from the goodness of fit tests and values of the parameters (constants) associated with the various correlations. Table VI-3 shows MSE values and percentage of data points

Table VI-3. Goodness of fit and parameters for hold-up correlations based on dimensionless groups (using data for all waxes; ϵ_g - %).

Correlation 1:

$$\epsilon_g = k_1(Bo)^{k_2}(Ga)^{k_3}(Fr)^{k_4}(We)^{k_5}$$

Correlation 2:

$$\frac{\epsilon_g}{100 - \epsilon_g} = k_1(Bo)^{k_2}(Ga)^{k_3}(Fr)^{k_4}(We)^{k_5}$$

	ORIFICE	TYPE	SMP[a]	
Foam	Yes	No	Yes	No
No. of points	175	349	61	36
MEAN SQUARE ERROR:				
Correlation 1	26.9	7.85	158	–[b]
Correlation 2	28.8	7.89	182	–[b]
PERCENTAGE OF POINTS WITHIN ±30%:				
Correlation 1	86.9	90.8	86.9	–[b]
Correlation 2	82.9	91.7	88.5	–[b]
PARAMETERS FOR CORRELATION 1:				
k_1	11.9	23.7	0.05	–[b]
k_2	-0.19	0.15	1.20	–[b]
k_3	0.11	0.01	-0.03	–[b]
k_4	0.19	0.62	0.14	–[b]
k_5	0.13	-0.01		
PARAMETERS FOR CORRELATION 2:				
k_1	0.06	0.35	3.04×10^{-8}	–[b]
k_2	-0.41	0.17	2.53	–[b]
k_3	0.23	0.01	0.10	–[b]
k_4	0.18	0.74	0.23	–[b]
k_5	0.22	-0.01		

[a] Weber number (We) was not included in correlations for this distributor.

[b] Parameters could not be estimated from these data due to convergence problems.

within the ±30% error band for correlations based on dimensional analysis. Problems with convergence prevented the correlation of hold-up data obtained using SMP distributors in the "slug flow" regime. Hold-up values obtained using orifice type distributors in the "slug flow" and "churn-turbulent" regimes have excellent reproducibility (see Section V-B.2.), therefore, the MSE for these data is significantly lower than for the other data groups. Greater than 90% of these data are within ±30% of the predicted values. These results also show that both correlations give similar fits for the different data groups, although the first correlation gives marginally lower MSE values than the second correlation. Table VI-4 summarizes goodness of fit tests for the two empirical correlations. In general the MSE values with these correlations are higher than those from correlations based on dimensional analysis. This would be expected since the empirical correlations have fewer parameters (constants) and therefore lower degrees of freedom. Table VI-5 shows goodness of fit test results for literature correlations. Convergence could not be obtained when Akita and Yoshida's correlation was used with three of the four data groups. For hold-up data in the "slug flow" and "churn-turbulent" regimes, obtained using orifice type distributors, this correlation gave a significantly higher MSE compared to other correlations and the fit was relatively poor. As expected, Bach and Pilhofer's correlation provides an excellent fit for data in the "slug flow" and "churn-turbulent" regimes.

Based on the lowest MSE values, the correlations that best describe hold-up data from the present study are presented here.

Table VI-4. Goodness of fit and parameters for empirical hold-up correlations (using data for all waxes; ϵ_g - %).

Correlation 1:

$$\epsilon_g = k_1 u_g^{k_2}$$

Correlation 2:

$$\epsilon_g = \frac{k_1 u_g}{1 - k_2 u_g}$$

	ORIFICE	TYPE	SMP	
Foam	Yes	No	Yes	No
No. of points	175	349	61	36
MEAN SQUARE ERROR:				
Correlation 1	50.4	8.17	162	16.4
Correlation 2	46.7	8.45	162	17.4
PERCENTAGE OF POINTS WITHIN ±30%:				
Correlation 1	70.3	90.8	86.9	94.4
Correlation 2	72.0	91.4	90.2	86.1
PARAMETERS FOR CORRELATION 1:				
k_1	93.7	84.3	105.9	98
k_2	0.41	0.59	0.15	0.61
PARAMETERS FOR CORRELATION 2:				
k_1	1425	466	10560	409
k_2	29.44	11.59	130	6.82

Table VI-5. Goodness of fit and parameters for hold-up correlations from literature (using data for all waxes; ϵ_g - %).

Correlation 1 (Akita and Yoshida):

$$\frac{\epsilon_g}{(100 - \epsilon_g)^4} = k_1(Bo)^{k_2}(Ga)^{k_3}(Fr)^{k_4}$$

Correlation 2 (Bach and Pilhofer):

$$\frac{\epsilon_g}{(1 - \epsilon_g)} = k_1 \left[\frac{u_g{}^3 \rho_\ell{}^2}{\mu_\ell g(\rho_\ell - \rho_g)}\right]^{k_2}$$

	ORIFICE	TYPE	SMP	
Foam	Yes	No	Yes	No
No. of points	175	349	61	36
MEAN SQUARE ERROR:				
Correlation 1	–[a]	32.00	–[a]	–[a]
Correlation 2	46.5	8.14	185	17.1
PERCENTAGE OF POINTS WITHIN ±30%:				
Correlation 1	–[a]	75.2	–[a]	–[a]
Correlation 2	71.4	91.7	88.5	94.4
PARAMETERS FOR CORRELATION 1:				
k_1	–[a]	3.85X10^5	–[a]	–[a]
k_2	–[a]	0.14	–[a]	–[a]
k_3	–[a]	0.08	–[a]	–[a]
k_4	–[a]	1.02	–[a]	–[a]
PARAMETERS FOR CORRELATION 2:				
k_1	0.321	0.128	2.174	0.133
k_2	0.19	0.24	0.08	0.27

[a] Parameters could not be estimated from these data due to convergence problems.

Group 1 - Orifice type distributors in the "foamy" regime:

$$\epsilon_g\ (\%) = 12(Bo)^{-0.19}(We)^{0.13}(Ga)^{0.11}(Fr)^{0.19} \qquad 0.01 \leq u_g \leq 0.07\ m/s \qquad (VI\text{-}14)$$

The above correlation for hold-up in the "foamy" regime shows that hold-up tends to increase with an increase in the Weber number (i.e. the jet velocity). This is as expected since our results show that hold-up in the "foamy" regime increases as hole size of the distributor is decreased (see Section V-3.5.). It was also shown earlier (see Section V-B.4.) that hold-up in the larger column (0.229 m ID) was lower than hold-up in the smaller column (0.051 m ID) in the "foamy" regime. In the above correlation, column diameter (d_c) is associated with the Bond number (Equation (VI-2)), Galileo number (Equation (VI-4)) and the Froude number (Equation (VI-5)). This implies that the gas hold-up is proportional to $d_c^{-0.15}$, i.e. hold-up decreases as column diameter increases. Figure VI-2 shows the parity plot for this correlation. In general, predicted hold-up values are in fairly good agreement with experimental data.

Group 2 - Orifice type distributors in the "slug flow" and "churn turbulent" regimes:

$$\epsilon_g\ (\%) = 24(Bo)^{0.15}(We)^{-0.01}(Ga)^{0.01}(Fr)^{0.62} \qquad 0.01 \leq u_g \leq 0.15\ m/s \qquad (VI\text{-}15)$$

A comparison of Equation (VI-15) with Equation (VI-14) shows that in the "slug flow" and "churn-turbulent" regimes, the Weber number does not have a significant effect on the gas hold-up. This implies that the type of distributor used has no effect on the gas hold-up. This is in agreement with results presented earlier (see Section V-2.5.). The terms involving the Bond number, Galileo number and the Froude number can be simplified to

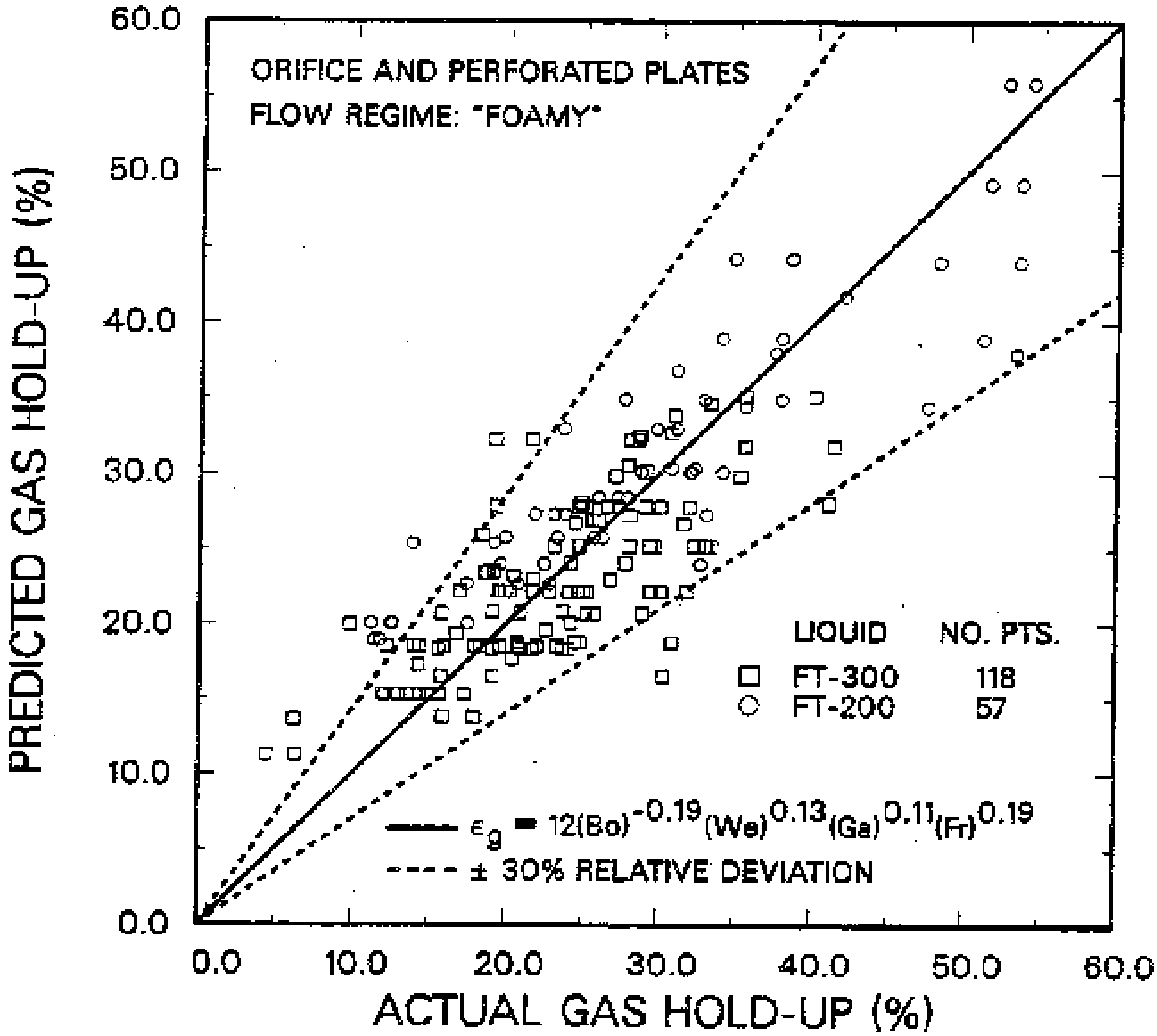

Figure VI-2. Comparison between experimental and predicted gas hold- ups

show that column diameter has no effect on the average gas hold-up. This is once again as expected and is in agreement with results presented earlier (see Section V-B.4.). Equation (VI-15) shows a weak dependence of hold-up on the Weber number and the Galileo number, therefore, the parameters (constants) for the correlation were reevaluated after dropping off these two groups. The resulting equation is:

$$\epsilon_g\ (\%) = 25(Bo)^{0.15}(Fr)^{0.60} \qquad 0.01 \le u_g \le 0.15\ m/s \qquad (VI\text{-}16)$$

This equation also shows that column diameter has no effect on the average gas hold-up. The simplified equation increased the MSE from 7.85 (for Equation (VI-15)) to 7.90, however, the percentage of points within ±30% of the predicted values remained the same for the two equations. A parity plot for this correlation is shown in Figure VI-3. The agreement between the predicted and measured values of gas hold-up is very good, with 91% of the data within ±30% of the predicted values.

Group 3 - SMP distributor in the "foamy" regime:

$$\epsilon_g\ (\%) = \frac{10560 u_g}{1+130 u_g} \qquad 0.01 \le u_g \le 0.12\ m/s \qquad (VI\text{-}17)$$

Hold-up values with the SMP distributor, in the presence of foam, remained fairly constant around 70% for velocities greater than 0.02 m/s. For gas velocities of 0.01 and 0.02 m/s hold-up varied between 20 and 60 %. Therefore, it is expected that an equation of the type shown above (Equation (VI-17)) would best describe this type of behavior. A parity plot for this correlation is shown in Figure VI-4.

Group 4 - SMP distributor in the "slug flow" regime:

$$\epsilon_g\ (\%) = 98 u_g^{0.61} \qquad 0.01 \le u_g \le 0.12\ m/s \qquad (VI\text{-}18)$$

A parity plot for this correlation is shown in Figure VI-5. Majority of

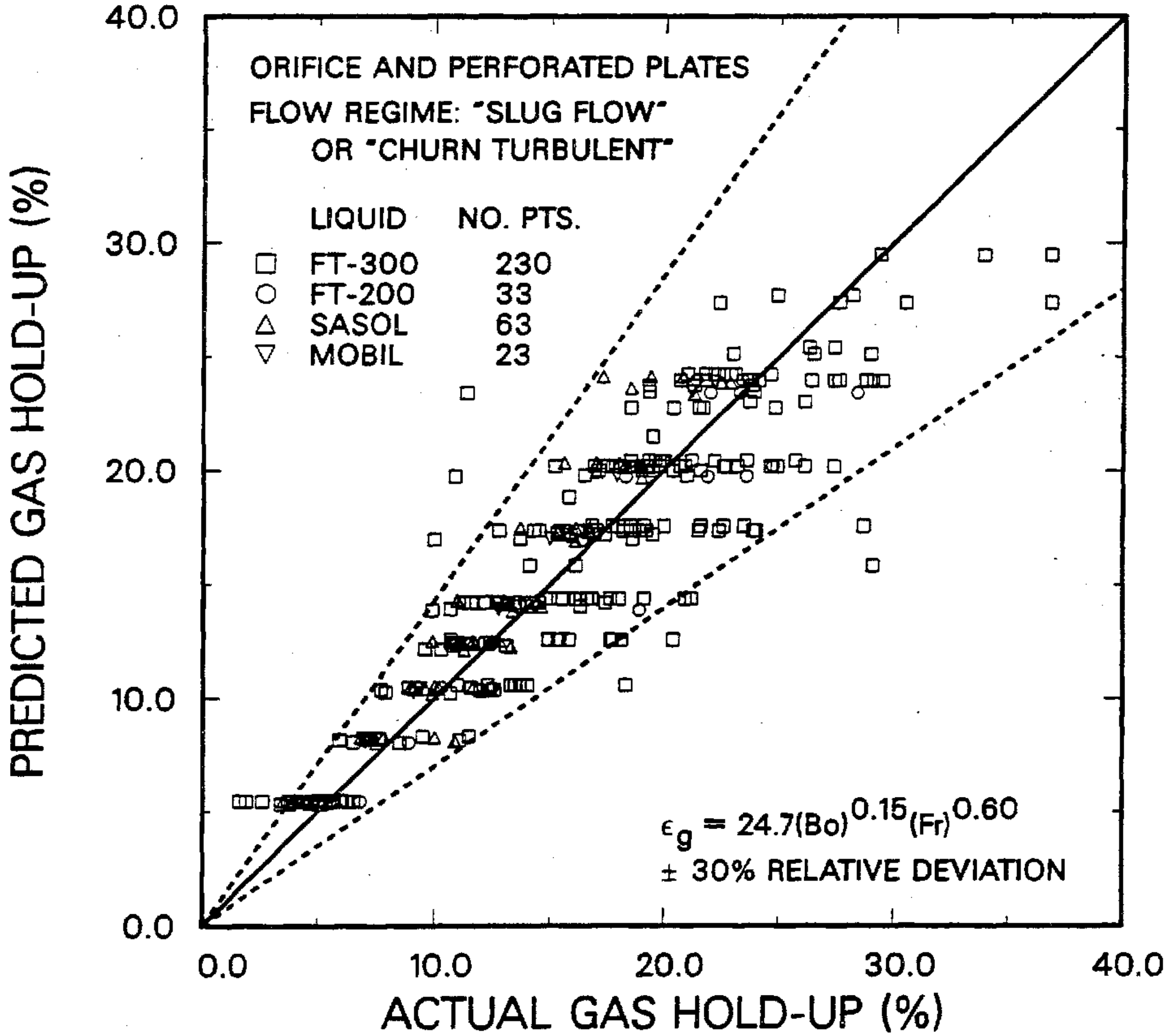

Figure VI-3. Comparison between experimental and predicted gas hold- ups

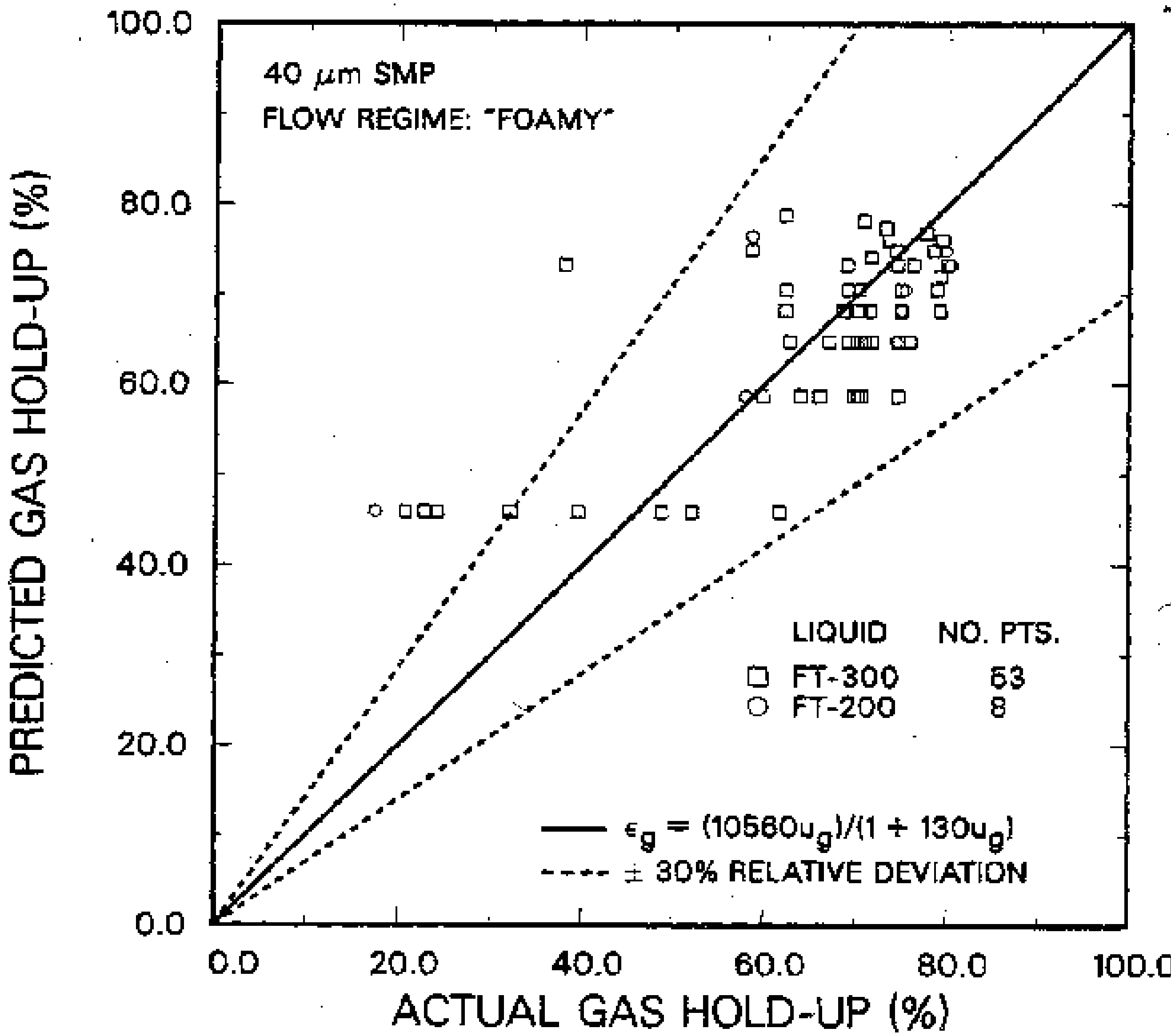

Figure VI-4. Comparison between experimental and predicted gas hold- ups

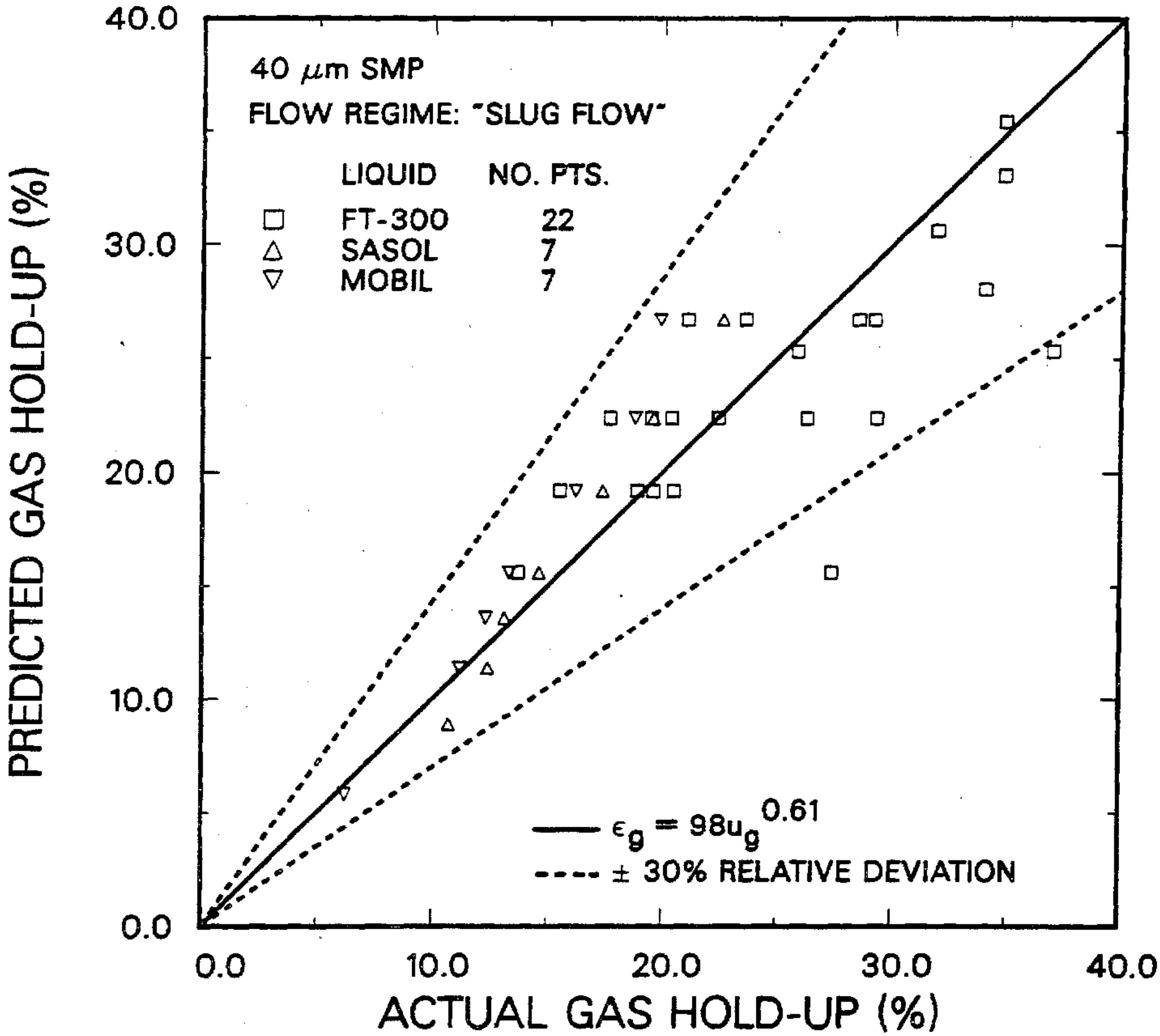

Figure VI-5. Comparison between experimental and predicted gas hold- ups

these data were obtained from runs conducted with reactor waxes at 265°C and hold-up appeared to be affected by gas velocity alone. The correlation presented in Equation (VI-18) provides the best description of these data.

B.6. Comparison with Literature

Bach and Pilhofer's correlation provides an excellent fit for average gas hold-up data in the "slug flow" and "churn turbulent" regimes (Figure VI-1). The constants in this equation were reevaluated and the new equation is:

$$\frac{\epsilon_g}{100 - \epsilon_g} = 0.128 \left[\frac{u_g^3 \rho_\ell^2}{\mu_\ell(\rho_\ell - \rho_g)} \right]^{0.24} \qquad 0.01 \le u_g \le 0.15 \text{ m/s} \qquad \text{(VI-19)}$$

The new coefficient is 0.128 compared to 0.115 for the original equation and the new exponent is 0.24 compared to 0.23 for the original equation. The new values compare very well with those for the equation proposed by Bach and Pilhofer. Hold-up data for FT-300 wax obtained at 265°C in the "slug flow" and "churn turbulent" flow regimes were also correlated separately and the constants for Bach and Pilhofer's equation reevaluated. These values are 0.140 and 0.23 respectively, and are essentially the same as the ones in the Bach and Pilhofer correlation.

Deckwer et al. (1980) used an empirical equation to correlate hold-up values obtained using molten paraffin wax, which is given by

$$\epsilon_g\ (\%) = 840 u_g^{1.1} \qquad u_g < 0.04 \text{ m/s} \qquad \text{(VI-20)}$$

The above equation is based on measurements made using gas velocities less than 0.04 m/s. Recently Sanders et al. (1986) obtained hold-up values with FT-300 wax for gas velocities up to 0.06 m/s and found that their data could be described with Equation (VI-20). When data obtained in the present study in the "foamy" regime were correlated with a similar equation, the following equation was obtained

$$\epsilon_g\ (\%) = 94u_g^{0.41} \qquad 0.01 \le u_g \le 0.07\ \text{m/s} \qquad \text{(VI-21)}$$

for orifice type distributors and

$$\epsilon_g\ (\%) = 106u_g^{0.15} \qquad 0.01 \le u_g \le 0.12\ \text{m/s} \qquad \text{(VI-22)}$$

for the 40 μm SMP distributor. When data in the "slug flow" and "churn turbulent" flow regime were correlated the following equation was obtained

$$\epsilon_g\ (\%) = 84u_g^{0.59} \qquad \text{(VI-23)}$$

for orifice type distributors and

$$\epsilon_g\ (\%) = 98u_g^{0.61} \qquad \text{(VI-24)}$$

for the 40 μm SMP distributor. The equation proposed by Deckwer et al. (Equation (VI-20)) shows that hold-up increases almost proportionally with superficial gas velocity, while results from the present study showed that hold-up values tend to level off at higher gas velocities. A possible explanation for the discrepancy is the range of velocities employed in the two studies. Equation (VI-20) is based on measurements made for gas velocities less than 0.04 m/s. For this range of gas velocities, hold-up values (in the presence of foam) tend to increase proportionally with gas velocity as was observed in the present study. Similar observations were also made by Kuo et al. (1985) in their studies with FT-200 wax. The correlation used to fit some of their data for gas velocities less than 0.06 m/s is

$$\epsilon_g\ (\%) = 1030u_g^{1.1} \qquad u_g \le 0.06\ \text{m/s} \qquad \text{(VI-25)}$$

The correlations developed in the present work are applicable to a greater range of gas velocities than the existing correlations for molten waxes. It is obvious from the present study that a single correlation cannot be used to predict hold-up data, since hold-up is significantly affected by the presence or absence of foam. The correlations presented above are based on a large number of data points, therefore, it is expected that they describe the hold-up behavior of molten waxes fairly well. For the mode of operation in the "foamy" regime, with orifice and perforated plate distributors, Weber number and column diameter have a significant effect on the gas hold-up, therefore a correlation that accounts for these effects is necessary for predicting the average gas hold-up values. Equation (VI-14) provides a good fit of data in this regime. In the "slug flow" and "churn-turbulent" regimes, hold-up is independent of column diameter and distributor type. Equation (VI-16) should be used to predict hold-up values in this flow regime. Hold-ups with the SMP distributor are essentially functions of the gas velocity alone. In the "foamy" regime, the dependence on u_g is very weak at velocities greater than 0.02 m/s and Equation (VI-17) can be used to predict hold-ups under these conditions. In the "slug flow" regime, with the SMP distributor, a limited amount of data were available and they could be correlated with Equation (VI-18).

C. CORRELATIONS FOR SPECIFIC INTERFACIAL AREA

The dynamic gas disengagement technique allowed the estimation of the Sauter mean diameter at different superficial gas velocities during a given run. The Sauters along with the corresponding average gas hold-up values were then used to estimate specific interfacial area from

$$a = \frac{6\epsilon_g}{d_s} \qquad \text{(VI-26)}$$

The specific interfacial area values were divided into the following six groups:

Group 1. Data from experiments conducted with FT-300 wax at 200°C.