APPENDIX C

DEVELOPMENT OF EQUATIONS FOR DGD AND SAMPLE CALCULATIONS

1. Derivation of the Relationship Between Normalized Liquid Level and Time

The general equation for the dynamic hold-up, $\epsilon_g(t)$ in the discretized form is [Equation (V-8) in Section D.3]:

$$\epsilon_g(t) = \epsilon_{go} \sum_{i=1}^{k} f_i \left[1 - \frac{tu_{Bi}}{H} \right] \qquad t^*_{N-k+1} \geq t > t^*_{N-k} \qquad \text{(C-1)}$$

The following equations can be used to modify Equation (C-1):

$$\epsilon_g(t) = \frac{H-H_s}{H} \qquad \text{(C-2)}$$

$$\epsilon_{go} f_i = \epsilon_{goi} \qquad \text{(C-3)}$$

where H_s is the height of the ungassed liquid (static height). Equation (C-1) now becomes:

$$\frac{H-H_s}{H} = \sum_{i=1}^{k} \epsilon_{goi} - \frac{t}{H} \sum_{i=1}^{k} \epsilon_{goi} u_{Bi} \qquad \text{(C-4)}$$

multiplying both sides by H/H_o gives

$$\frac{H}{H_o} - \frac{H_s}{H_o} = \frac{H}{H_o} \sum_{i=1}^{k} \epsilon_{goi} - \frac{t}{H_o} \sum_{i=1}^{k} \epsilon_{goi} u_{Bi} \qquad \text{(C-5)}$$

Equation (C-5) yields, upon rearrangement:

$$\frac{H}{H_o} = \frac{H_s/H_o}{1 - \sum_{i=1}^{k} \epsilon_{goi}} - t \frac{\sum_{i=1}^{k} \epsilon_{goi} u_{Bi}}{H_o \left[1 - \sum_{i=1}^{k} \epsilon_{goi} \right]} \qquad t^*_{N-k+1} \geq t^* > t_{N-k} \qquad \text{(C-6)}$$

This is an equation of a straight line with the first term on the right hand side representing the intercept and the second term represents the slope multiplied by time (t).

2. <u>Derivation of the Expressions for ϵ_{goi} and u_{Bi}</u>

From Equation (C-6) the intercept for the k-th straight line is given as:

$$b_k = \frac{H_s/H_o}{1 - \sum_{i=1}^{k} \epsilon_{goi}} \qquad \text{(C-7)}$$

This can be rearranged to give:

$$\sum_{i=1}^{k} \epsilon_{goi} = 1 - \frac{H_s}{H_o b_k} \qquad \text{(C-8)}$$

Further rearrangement gives the fractional gas hold-up corresponding to the k-th bubble class:

$$\epsilon_{gok} = 1 - \frac{H_s}{H_o b_k} - \sum_{i=1}^{k-1} \epsilon_{goi} \qquad \text{(C-9)}$$

Similar manipulation of the expression for intercept of the k-1'th straight line would result in the following expression:

$$\sum_{i=1}^{k-1} \epsilon_{goi} = 1 - \frac{H_s}{H_o b_{k-1}} \qquad \text{(C-10)}$$

Combination of Equations (C-9) and (C-10) will then result in the following expression for ϵ_{gok}:

$$\epsilon_{gok} = \frac{H_s}{H_o} \left[\frac{1}{b_{k-1}} - \frac{1}{b_k} \right] \qquad \text{(C-11)}$$

From Equation (C-6) the slope of the k-th straight line is:

$$s_k = \frac{\sum_{i=1}^{k} \epsilon_{goi} u_{Bi}}{H_o \left[1 - \sum_{i=1}^{k} \epsilon_{goi} \right]} \tag{C-12}$$

This equation can be rearranged to give:

$$\sum_{i=1}^{k} \epsilon_{goi} u_{Bi} = s_k H_o \left[\sum_{i=1}^{k} \epsilon_{goi} - 1 \right] \tag{C-13}$$

Substitution of Equation (C-8) into Equation (C-13) results in:

$$\sum_{i=1}^{k} \epsilon_{goi} u_{Bi} = - \frac{s_k H_s}{b_k} \tag{C-14}$$

which upon further manipulation gives the expression for the rise velocity of bubbles belonging to the k-th class:

$$u_{Bk} = \left[- \frac{s_k H_s}{b_k} - \sum_{i=1}^{k-1} \epsilon_{goi} u_{Bi} \right] \frac{1}{\epsilon_{gok}} \tag{C-15}$$

Similar rearrangement of the expression for the slope of the k-1'th straight line yields

$$\sum_{i=1}^{k-1} \epsilon_{goi} u_{Bi} = \frac{s_{k-1} H_s}{b_{k-1}} \tag{C-16}$$

Combination of Equations (C-11), (C-15) and (C-16) gives the following expression for u_{Bk}:

$$u_{Bk} = \frac{H_o \left[b_k s_{k-1} - b_{k-1} s_k \right]}{b_k - b_{k-1}} \tag{C-17}$$

The general expressions for ϵ_{goi} and u_{Bi}, for $i = 1 \rightarrow N$ are

$$\epsilon_{goi} = \frac{H_s}{H_o} \left[\frac{1}{b_{i-1}} - \frac{1}{b_i} \right] \tag{C-18}$$

$$u_{Bi} = \frac{H_o \left[b_i s_{i-1} - b_{i-1} s_i \right]}{b_i - b_{i-1}} \tag{C-19}$$

In the above equations, b_o represents the normalized static height (i.e. $b_o = H_s/H_o$) and b_N represents the normalized expanded height at time t=0 (i.e. $b_N = H_o/H_o = 1$).

3. <u>Equations for Bimodal and Trimodal Distributions</u>

The equations for ϵ_{goi} and u_{Bi} developed above assumed N distinct bubble sizes. For the case of two bubble sizes the general equations (C-18 and C-19) reduce to the following:

$$\epsilon_{goS} = 1 - \frac{H_s}{b_1 H_o} \tag{C-20}$$

$$u_{BS} = - \frac{H_s s_1}{b_1 - \frac{H_s}{H_o}} \tag{C-21}$$

and

$$\epsilon_{goL} = \frac{H_s}{H_o} \left[\frac{1}{b_1} - 1 \right] \tag{C-22}$$

$$u_{BL} = \frac{H_o \left[s_1 - b_1 s_2 \right]}{1 - b_1} \tag{C-23}$$

where S and L indicate small and large bubbles.

For the case of three bubble sizes, i.e. small (S), medium (M) and large (L), the equations for small bubbles are the same as above (i.e. Equations (C-20) and (C-21)), the remaining equations are as follows:

$$\epsilon_{goM} = \frac{H_s}{H_o} \left[\frac{1}{b_1} - \frac{1}{b_2} \right] \tag{C-24}$$

$$u_{BM} = \frac{H_o \left[b_2 s_1 - b_1 s_2 \right]}{b_2 - b_1} \tag{C-25}$$

$$\epsilon_{goL} = \frac{H_s}{H_o} \left[\frac{1}{b_2} - 1 \right] \tag{C-26}$$

$$u_{BL} = \frac{H_o \left[s_2 - b_2 s_3 \right]}{1 - b_2} \tag{C-27}$$

The equations for the fraction of bubbles in a given class, the number of bubbles in a given class, and the Sauter mean diameter, are the same as those developed for N bubble classes (i.e. Equations (V-14), (V-25) and (V-27), respectively).

4. <u>Example and Sample Calculations</u>

Figure C-1 shows results for normalized liquid level (H/H_o) versus time elapsed (t) for FT-300 wax (Run 13-3). For this case, the data at u_g=0.01 m/s can be fitted by two straight lines (bimodal distribution), while data at u_g=0.3 and u_g=0.9 m/s can be fitted to three straight lines (trimodal distribution). t_1^* and t_2^* indicate times corresponding to the two break points for the curve at u_g=0.09 m/s. At time t_1^* all large bubbles have disengaged, and at time t_2^* all medium bubbles have disengaged from the liquid. The normalized liquid level data at 0.03 m/s show that medium size

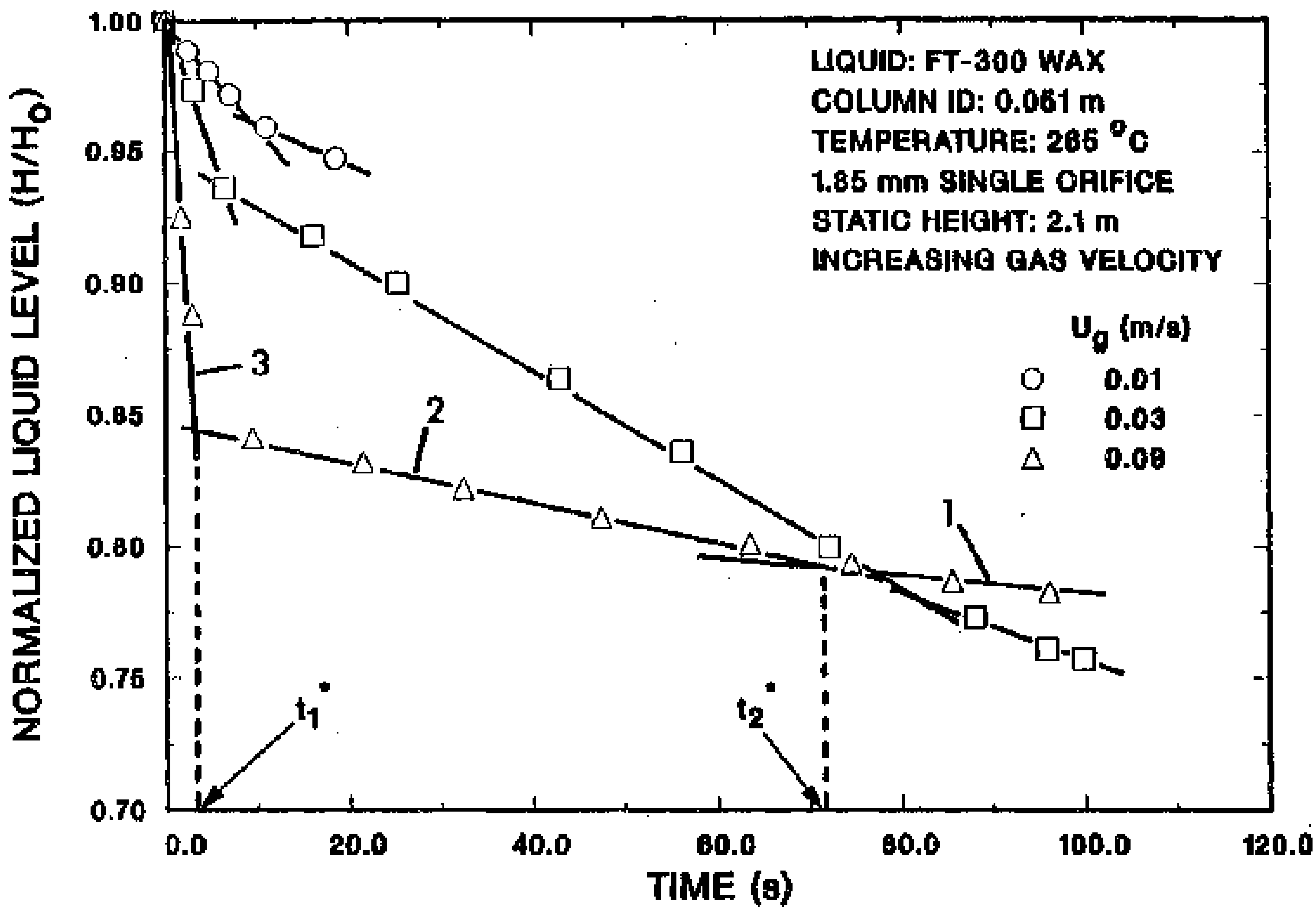

Figure C-1. Change in the normalized liquid level as a function of time and superficial gas velocity (Run 13-3) bubbles disengaged over a longer period of time at this velocity compared to 0.01 and 0.09 m/s because foam was present at 0.03 m/s and the rate at which the level dropped (including foam) was much slower.

Figures C-2 and C-3 are examples of bubble size distribution curves obtained from DGD measurements during Run 13-3. The figures show the variation of bubble size (small, medium and large) with superficial gas velocity. The size of small bubbles has a maximum at u_g=0.01 m/s and is fairly constant thereafter. This is typical of all runs (including those with reactor waxes). The medium size bubbles also show a similar behavior for most runs. However, large sized bubbles did not show a fixed trend. Their diameter increased with u_g for most runs. This increase was more pronounced for runs conducted in the 0.051 m ID column, probably due to the rapidly growing slugs; the increase was very gradual in the 0.229 m ID column, indicating that large bubbles had approached their maximum value.

The following calculations illustrate the procedure used to estimate the hold-ups corresponding to the various bubble size classes and the Sauter mean diameter. H/H_o versus t data from Run 13-3 at a superficial gas velocity of 0.09 m/s are used in this example. The data are plotted on Figure C-1 and were fitted to three straight lines (i.e. a trimodal distribution).

a. The first step is to determine the slopes and intercepts of the three straight lines fitted to the data obtained at 0.09 m/s.

	Line 1	Line 2	Line 3
slope (s_i)	−0.0004	−0.0007	−0.0320
intercept (b_i)	0.8225	0.8460	0.9906
No. of points	4	14	5
R^2	0.9959	0.9945	0.9823

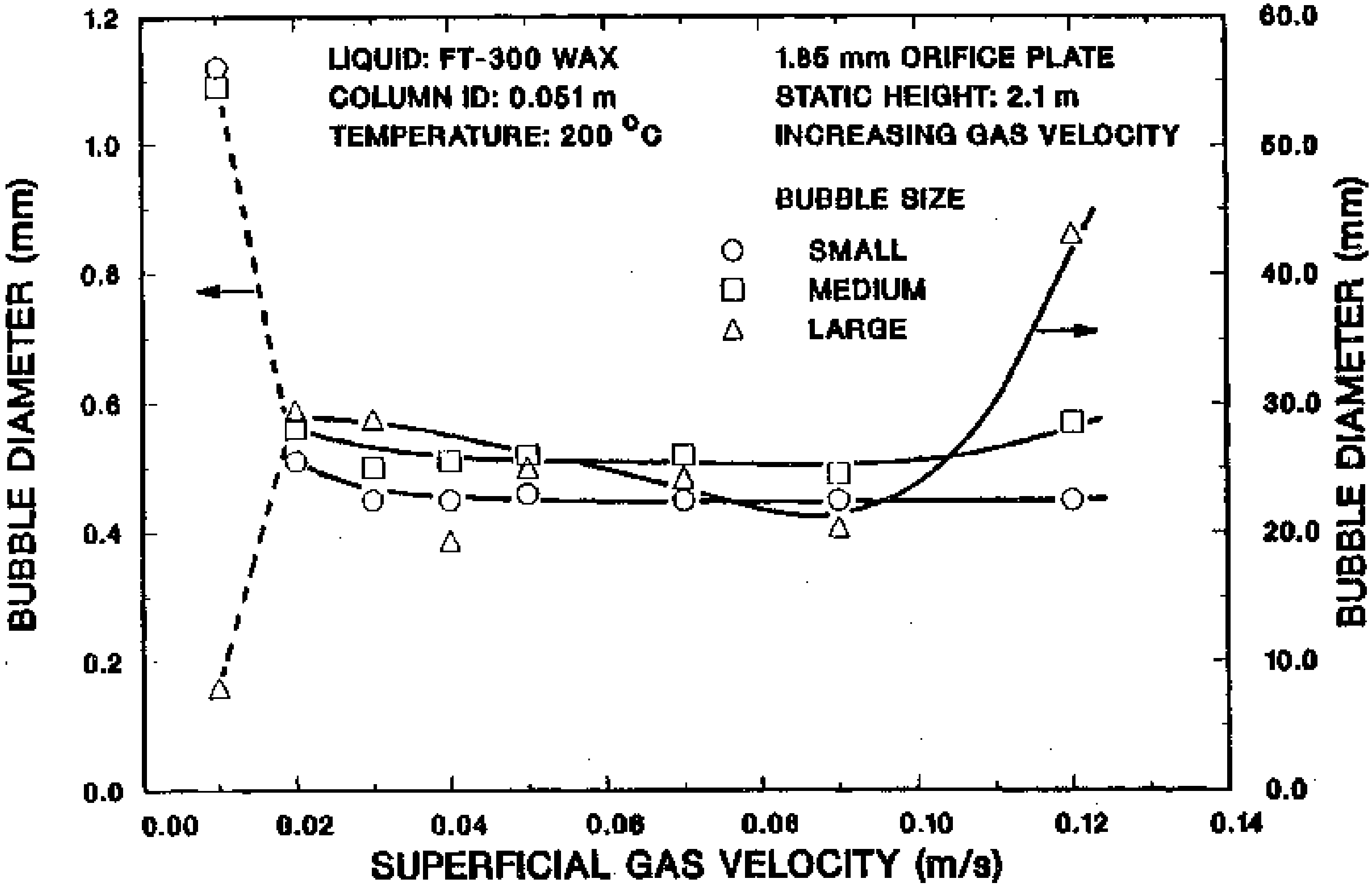

Figure C-2. Effect of superficial gas velocity on bubble sizes (Run 13-2)

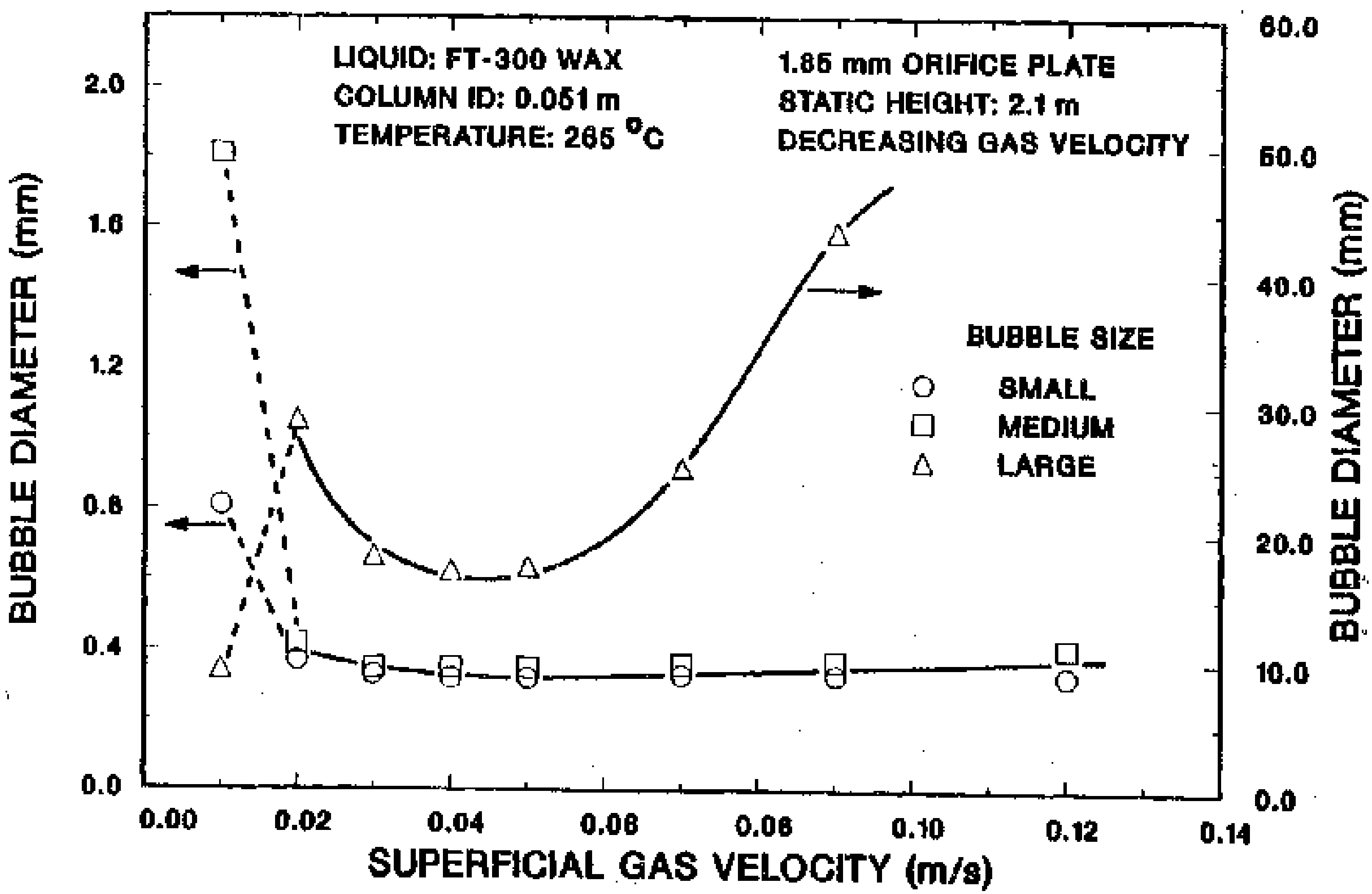

Figure C-3. Effect of superficial gas velocity on bubble sizes (Run 13-3)

Here, R^2 is the correlation coefficient and represents the goodness-of-fit of data to the straight line. A value of 1.0 would represent a perfect fit of data to the straight line. The values of R^2 for this case are close to 1.0 and imply a very good fit.

b. The values for ϵ_{goi} and u_{Bi} can now be estimated using Equations (C-20), (C-21), and (C-24) – (C-27). At 0.09 m/s, the static height (H_s) was 2.1 m and the expanded liquid height at time zero (H_o) was 2.718 m.

The fractional gas hold-up for small bubbles is calculated using Equation (C-20) as

$$\epsilon_{goS} = 1 - \frac{H_s}{b_1 H_o}$$

$$= 1 - \frac{2.1}{(0.8225)(2.718)}$$

$$= 0.0606 \text{ (or } 6.06\%)$$

Similarly, Equation (C-24) is used to calculate the fractional gas hold for medium size bubbles.

$$\epsilon_{goM} = \frac{H_s}{H_o}\left[\frac{1}{b_1} - \frac{1}{b_2}\right]$$

$$= \frac{2.1}{2.718}\left[\frac{1}{0.8225} - \frac{1}{0.8460}\right]$$

$$= 0.0261 \text{ (or } 2.61\%)$$

The fractional gas hold-up for large bubbles is calculated using Equation (C-26) as

$$\epsilon_{goL} = \frac{H_s}{H_o}\left[\frac{1}{b_2} - 1\right]$$

$$= \frac{2.1}{2.718}\left[\frac{1}{0.8460} - 1\right]$$

$$= 0.1407\ (14.07\%)$$

The rise velocity for the small bubbles is calculated using Equation (C-21) as

$$u_{BS} = -\frac{H_s s_1}{b_1 - \frac{H_s}{H_o}}$$

$$= -\frac{(2.1)(-0.0004)}{0.8225 - \frac{2.1}{2.718}}$$

$$= 0.0168\ m/s$$

Equation (C-25) is used to calculate the rise velocity of medium size bubbles

$$u_{BM} = \frac{H_o\left[b_2 s_1 - b_1 s_2\right]}{b_2 - b_1}$$

$$= \frac{2.718[(0.8460)(-0.0004) - (0.8225)(-0.0007)]}{0.8460 - 0.8225}$$

$$= 0.0275\ m/s$$

and, Equation (G-27) gives the rise velocity of large bubbles

$$u_{BL} = \frac{H_o(s_2 - b_2 s_3)}{1 - b_2}$$

$$= \frac{2.718[-0.0007 - (0.8460)(-0.0323)]}{1 - 0.8460}$$

$$= 0.4654 \text{ m/s}$$

c. The volume fractions for the small, medium and large bubbles can be calculated using Equation V-14. The average gas hold-up at this velocity was 22.74 (or $\epsilon_{go} = 0.2274$).

Thus,

$$f_i = \frac{\epsilon_{goi}}{\epsilon_{go}}$$

For small bubbles, the volume fraction is

$$f_S = \frac{0.0606}{0.2274} = 0.27$$

for medium size bubbles it is

$$f_M = \frac{0.0261}{0.2274} = 0.11$$

and, for large bubbles the volume fraction is

$$f_L = \frac{0.1407}{0.2274} = 0.62$$

d. The bubble sizes corresponding to the small, medium and large classes of bubbles can be estimated using the appropriate correlations. Figure V-65 shows the relation between bubble rise velicity and bubble diameter for FT-300 wax at 265°C. This plot can be used along with bubble rise velocities (u_{Bi}) estimated above to obtain the values for d_{Bi}.

Abou-el-Hassan's (1983) correlation (segment 1 on Figure V-65) was used to obtain:

$$d_{BS} = 0.33 \text{ mm}$$

as the size of small bubbles, and

$$d_{BM} = 0.37 \text{ mm}$$

as the size of medium bubbles.

The correlation proposed by Clift, Weber and Grace (1978; segment 2 on Figure V-65) was used to obtain the diameter of the large bubbles

$$d_{BL} = 43.45 \text{ mm}$$

e. Equation (V-27) is now used to estimate the Sauter mean bubble diameter for the bubble size distribution.

$$d_s = \frac{\epsilon_{go}}{\sum_{i=1}^{3} \epsilon_{goi}/d_{Bi}}$$

$$= \frac{0.2274}{\frac{0.0606}{0.33} + \frac{0.0261}{0.37} + \frac{0.1407}{43.45}}$$

$$= 0.88 \text{ mm}$$

f. The number of bubbles of a given size is calculated using Equation (V-25). The column diameter (d_c) for this example is 0.051 m.

$$n_i = \frac{3\epsilon_{goi} d_c^2 H_o}{2 d_{Bi}^3}$$

The number of small bubbles is

$$n_S = \frac{(3)(0.0606)(0.051)^2(2.718)}{(2)(0.33 \times 10^{-3})^3}$$

$$= 1.788 \times 10^7$$

The number of medium size bubbles is

$$n_M = \frac{(3)(0.0261)(0.051)^2(2.718)}{(2)(0.37 \times 10^{-3})^3}$$

$$= 5.764 \times 10^6$$

and the number of large bubbles is

$$n_L = \frac{(3)(0.1407)(0.051)^2(2.718)}{(2)(43.45 \times 10^{-3})^3}$$

$$= 18$$