In the WINDOW program when a bubble was found at the upper probe within the window frame, the following equation was executed:

$$mult = \frac{[S_L - S_U] + [E_L - E_U]}{2}$$

where mult = time delay for individual bubble

S_L = starting time of bubble on lower probe

S_U = starting time of bubble on upper probe

E_L = ending time of bubble on lower probe

E_U = ending time of bubble on upper probe

To make sure that the bubble on the upper probe was not computed with another bubble on the lower probe, a string condition was placed on it to disallow the bubble from further computations. When there was not a bubble on the upper probe within the time frame, a new starting time for a bubble on the lower probe was found and the process repeated itself. While Serizawa et al, (1974) only used the starting times, the ending times on the probes were used in this research to help compensate for the under and overestimations demonstrated in Figure 2.13. All the time delays for the individual bubbles were totaled and averaged to find the actual time delay between the probe tips.

2.4 EXPERIMENTAL RESULTS

The results found for the local void fraction were reliable. In comparison with the expansion of the mixture upon aeration; the local void fraction was underestimated by only 8-15% as shown in Table 2.1. With this small error, the threshold technique described above was taken to be reliable, since some underestimation might be expected due to deflection of bubbles from the probe.

Void fraction profiles were measured across the column diameter for the following conditions: at 1, 2 and 3 feet of unaerated water in the column, at

4.6, 7.3 and 10.4 scfm air flow, and at various heights from 6 inches above the distributor to just below the mixture surface. Results are shown in figures 2.14 to 2.40.

From observing the profiles near the distributor plate, it can be seen that the profile is somewhat flat. This type of distribution at the plate was to be expected because the air was introduced into the column evenly. However, as the probe was inserted at each height above 6", the profiles began to peak more near the center of the column. In the gulfstreaming effect, the liquid recirculates near the column wall. There is a radial inward liquid velocity above the plate forcing the bubbles inward (Freedman & Davidson, 1969). This results in a higher local void fraction near the center of the column. Again, the results back up this premise. Another relevant point about the reliability of the probes was the fact that at any location in the column, the local void fraction between the two probes transversing two diameters normal to one another was rarely over 10% in difference.

The bubble velocity results were less consistent than the void fraction results. At times, when the probe was away from the center, the time constants found from cross-correlation varied from 0.0005 to .0125 seconds which implied bubble velocites of 750 to 30.61 in/s, which was inconceivable. As seen from the experimental trace of the two probe signals in Figure 2.41, there is not a consistent pattern of bubbles impacting the lower probe then proceeding on to the upper probe. Poor results were ascribed to the bubble's radial motion which was not taken into consideration. Generally when the probe was located away from the center of the column, where a majority of the bubbles have some radial motion, the results found were poor relative to data taken at the column center. The bubble velocities found at the center of the column at 6" and 12" above the plate at three air flowrates are compared in Table 2.2 to the predicted bubble velocity found by using the one dimensional model of Clark et al. (1987) which is described later in this section. The table shows that the cross-correlation technique and the predicted bubble velocity varied from 5-25%.

The WINDOW program results proved less reliable than the cross-correlation technique. The problem with the program was that the window frame for a bubble to pass the upper probe affected the overall results. Figure 2.42 shows a histogram of individual bubble velocities in the center of the column at 12" above the plate and an air flowrate of 4.6 CFM. From these figures, it can be seen that the majority of the bubbles were found at the high velocity. Also, expected histogram should mimic a bell shaped curve with the best choice at the peak of the curve. Figure 2.42 does not demonstrate this.

Table 2.3 shows the number of bubbles striking the probes over 8 seconds at 12" above the plate and at the three different flowrates. This table concurs with the voidage data and demonstrates that the majority of the bubbles are located near the center while bubbles are scarce at the column wall.

2.5 ONE DIMENSIONAL MODELING

This section discusses two aspects of one dimensional modeling. Firstly, a computer implementation of the model of Clark et al. (1987) is presented since this was the model used to process the results presented above. Secondly, novel research relating the drift-flux model to bubble columns is discussed.

An important aspect of this research was to model the liquid velocity from the void fraction data. Early researchers (Rietema & Ottengraf, 1970; Hills, 1974) modeled the liquid velocity by a momentum balance, but viscosity or eddy viscosity was used in the model. However, in this research, mixing length theory was used to model the liquid velocity.

In the present research, the column's four inch radius was divided into 500 evenly spaced intervals. Since in the experimental work time-averaged local void fractions in the column were measured an inch apart, the voidages were divided into four sections of 125 intervals at 0.008 inch per interval. The inner voidages were computed by finding the slope between the end points. With the voidage profile computed, a density profile for the mixture of air and water in the column was calculated based on the local void fraction by the equation

$$\rho(r) = \rho_L (1 - \varepsilon(r)) + \rho_G \varepsilon(r) \qquad 2.1$$

where $\rho(r)$ = mixture density of radius r

ρ_L = liquid density

ρ_G = gas density

The second term of the righthand side is often neglected due to the low density of air. The average mixture density, ρ_{ave}, and the average mixture densities $\rho_i(r)$, within central cylinders of radius r were calculated by the equation:

$$\rho_i(r) = \frac{1}{i}\sum_{i=1}^{r} \rho(r) \qquad 2.2$$

This is acceptably accurate in the case of a flattened void profile, although strictly the more exact approach used by Clark et al. (1987) is to be preferred. At the time of writing data are being reprocessed using the exact approach. The shear stress profile was computed next using the force balance equation.

$$T(r) = T_w \left[1 + Rg \left(\frac{\rho_{ave} - \rho_i(r)}{2T_w} \right) \right] \left(\frac{r}{R} \right) \qquad 2.3$$

where T_w = wall shear stress (an unknown).

The shear stress may also be related to the velocity gradient in the column.

$$T(r) = T_L(r) + T_T(r) \qquad 2.4$$

where $T_L(r)$ = laminar shear stress

$T_T(r)$ = turbulent shear stress

The laminar shear stress equation is:

$$T_L(r) = \mu \frac{du(r)}{dr} \qquad 2.5$$

where μ = viscosity

$\frac{du(r)}{dr}$ = liquid velocity gradient

The turbulent shear stress is given by

$$T_T(r) = -\rho(r)\, L^2(r) \left| \frac{du(r)}{dr} \right| \frac{du(r)}{dr} \qquad 2.6$$

where L(r), the mixing length, is given by Clark et al., (1987),

$$L(r) = R \left[.14 - .08 \left(\frac{r}{R} \right)^2 - .06 \left(\frac{r}{R} \right)^4 \right] \qquad 2.7$$

From these equations one can find $\frac{dv}{dr}$ as a function of r for an assumed value of T_w. At the column wall there is the boundary condition that liquid velocity must be zero, so that the axialliquid velocity distribution across the column is readily found.

An.additional equation required before the liquid velocity profile can be computed is derived from the fact that the net flowrate of liquid across a cross-section of the column must be zero for a batch process column.

$$Q = 2\pi \int_0^R V_L(r) r (1-\epsilon(r)) dr = 0 \qquad 2.8$$

These equations were solved to yield the liquid velocity distribution in the following way. A value for wall shear was assumed, and the shear stress distribution across the column was computed. From the shear stress, the velocity gradient, and then the liquid velocity were found. If the net flowrate of liquid across the column cross-section was then not zero, a new value of wall shear stress was assumed, and the process repeated. When the net flow of liquid was zero, the correct liquid velocity profile had been found. Figure 2.42 shows the calculation scheme. Figures 2.44 through 2.46

show computed profiles of the local void fraction, mixture density, and the shear stress with one foot of water in the column, the probes were located at 6" above the distributor plate, and an air flowrate of 4.6 CFM. Figures 2.47 to 2.73 show velocity profiles computed in this way from the voidage data. Table 2.2 shows that centerline velocities found in this way agree acceptably with experimental data.

A second model was developed to predict the holdup in bubble columns with either zero or low liquid throughout using the drift-flux approach. Zuber and Findlay (1964) have presented an exhaustive derivation for the drift-flux model, which is based on the argument that holdup in a two-phase flow (typically bubble, slug or churn gas-liquid flow) is influenced by two separate phenomena. Firstly, it is acknowledged that the gas rises locally relative to the liquid due to phase density differences, a fact which may often be neglected in high velocity flows. Secondly, where a velocity distribution exists in the pipe, and where the gas is inhomogenously distributed across the pipe diameter, the gas may concentrated in a faster or slower region of flow, thus affecting the average gas holdup. The model is usually presented in the form

$$\frac{\bar{W}_g}{\bar{\epsilon}} = C_o \,(\overline{W_g + W_l}) + \frac{\overline{U_{gm}\,\epsilon}}{\bar{\epsilon}} \qquad 2.9$$

where W_g is gas volumetric flux (superficial velocity), W_l is liquid volumetric flux, ϵ is void fraction, U_{gm} is relative velocity between the gas and the <u>mixture</u> (i.e. $U_{gm} = U_g - (W_g + W_l)$, where U_g is gas velocity) and an overscore denotes an average over the column cross-section. The profile constant C_o is given by

$$C_o = \frac{\overline{\varepsilon\ (W_g + W_l)}}{\bar{\varepsilon}\ \overline{(W_g + W_l)}} \qquad 2.10$$

and is a measure of the interaction of the void and velocity distributions. Where gas is more concentrated in the faster region of flow, $C_o > 1$, and C_o is often taken as 1.2 for fast upward bubble flows, as shown in Table 2.4 below.

The term $\overline{U_{gm}\varepsilon}/\bar{\varepsilon}$ is the weighted average drift velocity, accounting for local slip. Since it has been shown that relative phase (bubble rise) velocity U_{GL} ($=U_G - U_L$) varies little over the pipe diameter (Serizawa et al., 1975), and since $U_{gm} = U_g\ (1-\varepsilon)$, with $\varepsilon < 0.25$ in bubble flow, the weighted average drift velocity is often taken as the rise velocity of a bubble in an infinite medium, U_∞. Experimental data generally support this simplification.

The drift-flux model has not remained inviolate. There is a mounting body of data to show that either C_o or $\overline{U_{GM}\varepsilon}/\bar{\varepsilon}$ cannot be taken as constant over an operating range. For example, Hills (1976), using a 0.15m pipe, acquired data which led to the development of a modified drift-flux model Also Shipley (1984), working with a "toy tower" circulation loop of 0.457m diameter, found his data best correlated by the dimensional equation.

$$\frac{\bar{W}_g}{\bar{\varepsilon}} = 1.2\ (Wg + W_L) + 0.24\ (m/sec) + 0.35 \left(\frac{\bar{W}_G}{\bar{W}_G + \bar{W}_L} \right)^2 (gD\bar{\varepsilon})^{1/2} \qquad 2.11$$

and some physical argument was provided for this approach. Clark and Flemmer (1985), using a 100mm diameter forced circulation loop in bubble flow, preferred to vary C_o as a function of void fraction, since C_o was found to be near 0.95 at low void fractions and near 1.2 when $\varepsilon \approx 0.2$. Subsequently Clark and Flemmer (1986) fitted the same data to a drift-flux model with two profile constants

$$\frac{\bar{W}_G}{\bar{\epsilon}} = C_G \bar{W}_G + C_L \bar{W}_L + \frac{U_{G\infty} \epsilon}{\bar{\epsilon}} \qquad 2.12$$

with C_G = 1.95 and C_L = 0.93. Jones (1985) acquired data in a bubble column with draft tubes ranging from 44 to 146mm in diameter. A drift-flux model overestimated the circulation in this column rate unless C_o was set to very high values (2 to 5) (Clark and Jones, 1987). Further argument on the variation of C_o has been presented by Lorenzi and Sotgia (1978), while careful consideration of the definition of C_o in the light of experimentally measured void and velocity profiles presented by Galaup (1975) and Serizawa et al. (1975) also supports the argument that C_o cannot remain constant.

The bubble column represents the extreme case of bubble flow where net liquid velocity is zero or low and the gas simply bubbles up through the liquid. Generally buoyancy effects and gas maldistribution cause circulation of liquid in the column. Even when gas is evenly distributed over the column base, circulation can occur, and it is worth taking the time to consider the mechanism governing circulation startup, since this has not been discussed elsewhere.

Let us assume that a column with even gas distribution starts to operate in an "ideal" or one-dimensional mode, where the gas void distribution is even over the cross section at all heights in the column, so that the mixture is homogeneous and has the same density throughout the column. At this stage no gross liquid circulation would occur. Since most two phase systems involve some degree of turbulence or mixing, let us propose that at some time a few bubbles move toward the center of the column, at any height, so that the concentration of bubbles is suddenly slightly higher near the center. Consider the mechanistic sequence of events which may arise. The mean mixture density in the central region of the column is now lower than density in the outer annulus, so that the hydrostatic head is greater over the height of the annulus than over the same height of the central core. Both the annular and central regions have the same pressure at the liquid surface, so that the difference in hydrostatic head causes a radial pressure difference deeper in the column (in fact, at every height below the point where the bubbles moved inward initially). This in turn causes an inward radial movement of liquid and initiates liquid circulation.

Bubbles rising from the distributor plate in the annular region are now moved inward a little by this radial inward liquid movement: the result is that bubble concentration near the center of the column increases and the liquid circulation increases. There is a positive feedback between the circulation and the radial bubble movement. Rapidly a stronger circulation pattern develops and the radial inward currents above the distributor plate continue to sweep more of the bubbles toward the center of the column (by analogy this is an inverted classifier).

One might expect that the circulation pattern would continue to increase indefinitely, but increasing upward velocity in the central region reduces bubble holdup in that region and hence the difference in hydrostatic heads tends to some limit. Driving force arising from the difference in heads is consumed by dissipation in the fluid (energy balance) or shear at the column walls (momentum balance).

The argument may also start by assuming that a small number of bubbles move initially into the annular region, in which case the inverse pattern is set up. Such patterns have been known to occur, especially in fluidized beds (Surma, 1986, Lin et al., 1985). In fact, this argument could be applied to any pattern, even multiple patterns in large tanks (Otero et al., 1985). However, it would seem that in the case of bubble columns and fluidized beds, there are some non-stochastic grounds for the selection of a specific pattern by initial bubble movement because at high gas velocities the circulation pattern is always upward at the center. Transition patterns have also been observed (Lin et al., 1985).

Consider a cross section of a tall bubbled column at half of the height of the mixture in the column: radial effects may be neglected. Let us pursue an analysis which commences with the same approach as the one-dimensional model derived above. The gas void distribution in the column can usually be well described by a relationship of the form

$$\varepsilon(r) = \varepsilon c\,[1 - (r/R)^{p}] \qquad 2.13$$

where εc is the voidage at the center of the column and R is the radius of the column. Figures 2.14 to 2.40 bear out this relationship.

The density, $\rho(r)$, as a function of position is given then given by:

$$\rho(r) = \rho_L(1 - \varepsilon(r)) + \rho_G\varepsilon(r) \qquad 2.14$$

where ρ_L is the liquid density, and where ρ_G, the gas density, is usually neglected. Substituting the relationship for voidage into equation 2.14:

$$\rho(r) = \rho_L\,[1 - \varepsilon c + \varepsilon c\,(r/R)^{p}] \qquad 2.15$$

Axial shear stress, T(r), may be found (Clark, et al., 1987) using the same force balance as previously employed

$$T(r) = \tau_w\,(r/R) + 1/2\, rg\,(\bar{\rho} - \rho_1\,(r)) \qquad 2.16$$

where g is gravitational acceleration and T_w is the unknown wall shear stress. $\rho - \rho_1\,(r)$ is the difference between average density over the whole radius, and average density within a radius, r, given by:

$$\bar{\rho} - \rho_L\,(r) = \rho_L\left(\frac{2\,\varepsilon c}{p + 2}\right)\left[1 - \left(\frac{r}{R}\right)^{p}\right] \qquad 2.17$$

For the fluid in the column, the rheological mixing length model which takes into account turbulent momentum transfer was again used. Shear stress was accordingly taken as:

$$\tau(r) = \mu\left(\frac{dU}{dr}\right) + L^2\rho\left|\frac{dU}{dr}\right|\left(\frac{dU}{dr}\right) \qquad 2.18$$

where μ is the viscosity, and L is the mixing length used above, in equation 2.7.

The liquid velocity profile, U(r), may be solved by equating shear stress in equations 2.16 and -2.18:

$$0 = T_w \left(\frac{r}{R} \right) + \frac{rg}{2} (\bar{\rho} - \rho i) - \mu \frac{du}{dr} - L^2 \rho \left(\frac{du}{dr} \right) \left(\frac{du}{dr} \right) \quad 2.19$$

Using an assumed wall shear stress the values for $\frac{du}{dr}$ and U can be found. Integrating U(r) (1 - ε) over the column cross-section then yields the liquid flux, W_L:

$$W_L = \frac{2}{R^2} \int_0^R U(r)\ (1 - \varepsilon)\ r\ dr \qquad 2.20$$

Note that local void fraction plays a role in this equation, whereas it was neglected in the simpler one dimensional analysis above. If W_L is equal to the desired liquid flux, then the assumed T_w was correct. For example, if no net liquid flow is desired (batch bubble column), T_w would have to be chosen until the integral in equation 2.20 assumed a value of zero. For the purpose of lucidity and efficiency, equation 2.19 may be made dimensionless, as follows:

$$0 = \left[N + \frac{Ga}{2} \frac{\varepsilon}{p+2} \right] r' - 2 \frac{dU'}{dr'} \left(\frac{\ell}{R} \right)^2 (1 - \varepsilon)\ Ga \left(\frac{dU'}{dr'} \right) \left| \frac{dU'}{dr'} \right| \quad 2.21$$

where:

$$N = \frac{T_w}{\mu} \sqrt{\frac{D}{g}} \qquad 2.22$$

$$U' = \frac{U}{\sqrt{gD}} \qquad 2.23$$

$$r' = \frac{r}{R} \qquad 2.24$$

And $Ga = \frac{\sqrt{gD}\ D\rho_L}{\mu}$ 2.25

which is a Galileo number: D is column diameter.

Equation 2.20 may be rearranged to give:

$$\frac{W_L}{\sqrt{gD}} = 2 \int_0^1 U' \ (1 - \varepsilon) \ r' \ dr' \qquad 2.26$$

In order to determine the drift flux parameter, C_o, it is necessary to find the average flux of the gas, W_G. The local gas volume flux is given by:

$$W_g = \varepsilon U + \frac{\varepsilon}{1 - \varepsilon} \ V_v \qquad 2.27$$

Hence:

$$W_g + W_\ell = U + \left(\frac{\varepsilon}{1 - \varepsilon}\right) V_v \qquad 2.28$$

And by definition of C_o:

$$C_o = \frac{\int_o^R \left[\varepsilon U + \left(\frac{\varepsilon^2}{1 - \varepsilon}\right) V_V \right] r \ dr}{\varepsilon_c \left(1 - \frac{2}{p+2}\right) \int_o^R \left[U + \left(\frac{\varepsilon}{1-\varepsilon}\right) V_v \right] r dr} \qquad 2.29$$

Or:

$$C_o = \frac{\int_o^1 \left[\varepsilon \ U' + \left(\frac{\varepsilon^2}{1-\varepsilon}\right) \ Fr \right] r' dr'}{\varepsilon_c \left(1 - \frac{2}{p+2}\right) \int_o^1 \left[U' + \left(\frac{\varepsilon}{1-\varepsilon}\right) Fr \right] r' dr'} \qquad 2.30$$

where the froude number, Fr, is given by:

$$Fr = \frac{V_v}{\sqrt{gD}} \qquad 2.31$$

The drift flux parameter thus depends on four variables, viz. Ga, Fr, ϵ_c, and ρ.

This circulation model was used to calculate the drift-flux parameter C_o for a wide range of column diameters, void distributions, fluid properties and local bubble rise velocities. Table 2.5 relates void distribution parameters to average voidage in the column. Figures 2.74 to 2.82 show how C_o varies with Ga, Fr and void distribution. It is interesting to note that C_o assumes very high values over most of the operating range, since the velocity profile is dictated entirely by buoyancy effects rather than by a net flow up the column (i.e. wall effects). This is evident in figure 2.83 which shows one of Hills' void profile with the sympathetic velocity distribution. This calculated velocity distribution agrees well with the velocity distribution measured by Hills using a Pavlov tube, which is a device that infers fluid velocity from the pressure field around a cylinder. Figure 2.84 serves to validate the model further with comparison of predicted centerline velocities measured by Hills for various gas flowrates.

The model was also used to find C_o when there is a net flow in the column. In figure 2.85 the curve for Fr = 0.25 corresponds to air-water upflow in a 100 mm diameter pipe with a typical air-water bubble rise velocity of 250 mm/sec. and with Re based on net upflow of liquid in the pipe. At high velocities it can be seen that C_o tends to a value between 1 and 1.5 which is what has been observed in practice (see table 2.4). However, at lower velocities buoyancy effects become significant with C_o assuming values as high as 3 or 4. This explains overestimation of void fraction and hence circulation rate in some air lift circulation devices (see comments by Clark and Jones, 1987). At a larger pipe diameter (Fr = 0.113) buoyancy effects play an even greater role, with a very high centerline velocity existing even at average velocities of 10 m/sec. One must conclude the C_o is highly variable in these circumstances so that a drift-flux model should be used with caution for low velocity or large diameter systems. Figure 2.86 illustrates some of the predicted velocity profiles which led to the construction of figure 2.87. The competing buoyancy and wall effects are evident here. Figure 2.88 yields C_o when the void fraction is a maximum at the pipe center over a broad range of liquid flow Reynolds numbers for a 500 mm diameter pipe (Fr = 0.113) for both upflow and downflow. A choking condition is evicert when the liquid downflow counteracts the upward rise of the bubbles, causing a void fraction to be present although $(W_g + W_\ell) = 0$. In consequence C_o assumes very high values.

It is well documented in the literature (Galaup, 1975; Serizawa et al., 1975; Nakoryakov et al., 1981) that saddle shape void profiles can also exist in low void fraction flows, and this is in keeping with the argument presented above for the "inverse" circulation pattern in bubble columns. Figures 2.88 show results as velocity profiles from the model for air-water upflow using the annular gas void distribution illustrated in figure 2.89. Buoyancy effects are so strong for the case of the 500mm pipe (Fr = 0.113) that the velocity profile remains saddle shaped (see figure 2.88) at all credible operating velocities. As a result C_o remains above unity, despite the fact that the bubbles are concentrated near the wall. However, for a 100mm pipe (Fr = 0.25), where wall effects are more significant, the computed velocity profile was similar to that in a single phase flow at higher velocities, so that C_o was driven to a value below unity.

One may conclude from this drift-flux analysis that in large diameter pipes C_o will always be greater than unity, assuming very high values at low net flowrates. However, in small diameter pipes C_o will still assume high values due to buoyancy effects at low velocities, but the value of C_o will depend strongly on the gas void distribution at higher velocities. When the void distribution is saddle-shaped, as occurs at low void fractions, C_o will be slightly below 1. At higher void fractions, where ε is a maximum at the center, C_o will assume typical values in the range 1.3 to 1.5. It can be seen that pipe size, flowrate and void fraction can all influence C_o, so that it is not surprising that many authors have found the necessity to modify drift-flux models to account for these problems. One must conclude that a direct drift-flux approach is not suitable unless the void distribution is known and that buoyancy effects are insignificant.

All of the velocity profiles generated above required prior knowledge of the void profile present in the column, so that a priori prediction of circulation is still not facilitated. However, by considering the arguments presented above on the origin of the circulation pattern, and by assuming all bubble motion to be causal (that is, caused only by gravity and average interphase drag) above the distributor, it should be relatively easy to develop an iterative model for the prediction of circulation in a vessel with even gas distribution, without having to assume bubble distribution across the diameter. Select, for the first iteration, an approximate circulation pattern in terms of the velocity distribution across a diameter half way up the column. Then, from continuity considerations, the average radial flux of liquid in the lower half is known at each value of the radius. Transport of bubbles towards the column center by the radial liquid flow in the entrance region can be determined quite accurately, since it is the total radial flux that determines the net bubble movement and the actual radial velocity need not be known as a function of height. The approach yields a gas flux and a voidage distribution half way up the column which can in turn be used to infer a liquid velocity distribution by applying the model described above: the calculation process is then repeated until the predicted velocity distribution agrees with he assumed velocity distribution. In other words we have two models to relate void distribution and velocity profile. One is the force balance model, used extensively above. The other involves the "classification" of bubbles which are moved radially as they rise from the distributor to the column center. When these two models agree on the relationship between void and velocity distribution, we might assume that this is a stable circulation solution. This is discussed in detail below, and an example follows.

Consider the zone in a bubble column just above the distributor plate, and assume that the column contains one large circulation cell, with liquid upflow at the center and downflow at the walls. In this zone there must accordingly be a radially inward flow of liquid. Bubbles rising from the distributor plate will be carried a radial distance inward before they rise to the center section of the column. Let us assume that this lower zone has height h (which is shown to be an arbitrary value) and that the radial velocity components are even over this height, see figure 2.90. Fluid delivered or removed axially from above to this lower zone will dictate the radial velocity profile. Let U(r) denote the axial velocity in the center zone and V(r) the radial velocity within the lower zone. From conservation of mass, neglecting the volume occupied by bubbles,

$$-2\pi hrV = -\int_r^R 2\pi rU \quad dr \qquad 2.32$$

which agrees with the fact that V must be zero at the column center and column wall. A bubble rising from the plate with a vertical velocity U_b will spend a time

$$t_z = h/U_b \qquad 2.33$$

within this zone, and during that time will be swept inwards with a velocity V(r), so that when it leaves the zone it will have moved inwards by a distance given by the integral

$$\int_{t=0}^{t_z} V(r)\, dt. \qquad 2.34$$

Practically (numerically!) one may trace the path of any bubble through this lower zone, assuming its lateral movement is governed causally by the liquid velocity, to find its position upon leaving the lower zone. For many bubbles, the gas flux $W_{gb}(r)$ at the base of the lower zone (at the distributor) may be translated into $W_g(r)$ in the central zone. Note that $W_g(r) = \varepsilon(r)\ (U_L(r) + V_v)$, in other words, the gas flux is the product of void fraction and gas velocity. This suggests the technique for determining circulation in a bubble column (or a pipe containing low velocity flows) with a knowledge only of distribution performance. The technique involves an assumption of $\varepsilon(r)$, followed by calculation of U(r). Hence V(r) may be calculated, and $W_{gb}(r)$ and V(r), $W_g(r)$ and then $\varepsilon(r)$ may be predicted. If this last void distribution is dissimilar from the one originally assumed, a new $\varepsilon(r)$ must be assumed and the procedure repeated. Presumably a stable solution results when a given $\varepsilon(r)$ (or W(r)) will predict the same $\varepsilon(r)$ (or W(r)), provided that the system can reach that stable condition upon startup. Perturbation studies could also be conducted in this way to assess how robust any particular circulation pattern will be.

An example of this new technique follows, based on the data of Hills. In his paper, data is presented for void and velocity distributions in a 138mm diameter bubble column operating with "Plate B", which by description should introduce gas evenly over the base of the column. From Hills' Figure 11, liquid velocity profiles were obtained for gas fluxes of 169 mm/sec and 38 mm/sec. From Hills' Figure 6, data on void fraction were obtained for the same gas throughputs. A computer program was written in Fortran 77 to translate the discrete values of velocity U(r) into radially inward velocities, V(r), in the lower zone above the distributor. This zone was assumed to be one column radius high, but it can be shown readily that this height is arbitrary (a "dummy variable") in this simplified analysis. The program then examined a flux of bubbles rising through this radial flow, being carried inward as they rose. In this way $W_g(r)$ was predicted from $W_{gb}(r)$, where $W_{gb}(r)$ was assumed to be an even flux with Hills' plate B.

Figure 2.91 shows the axial velocities in the center zone and radial velocities in the lower zone for h=R and for the gas flowrate of 38 mm/sec. Figure 2.92 shows the computed values of W(r) compared with the values of $W(r)$ (= ε (U_L + V_v)) found from Hills' paper. While agreement is not excellent, the trends are correct. Errors are ascribed to (i) inaccuracy in the data, (ii) the discrete and crude approach to finding V(r), (iii) the fact that the volume occupied by bubbles in the lower zone was not considered fully in this simplified model, (iv) the fact that bubble motion might not have injected air evenly and (v) the fact that the distributor plate might not have injected air evenly over the bed cross section. Considerable disagreement toward the column center also arises because errors in volumetric flows are mapped into very small area increments, yielding high errors in computed velocity in this region. Data for the 169 mm/sec gas flowrate were in greater error, most likely due to cause (iii) above.

Improving this approach will simply lead the researcher to use of a more rigorous fluid computational analysis, as discussed in section 4 of this report.