Effect of Liquid Medium

As mentioned previously, SASOL reactor wax and FT–300 wax behaved differently in the 0.05 m ID column. FT–300 wax has a tendency to foam, and as a result, gas holdups obtained with FT–300 wax were substantially higher than those obtained with SASOL reactor wax. Results from experiments with SASOL wax and FT–300 wax are shown in Figure 2.31. In particular, Figure 2.31a shows results from batch experiments conducted without solids and Figure 2.31b shows results for experiments conducted with 20 wt% 0 – 5 μm iron oxide particles at a superficial slurry velocity of 0.005 m/s. The results indicate that regardless of the presence of solids or liquid circulation, gas holdups are substantially higher with FT–300 wax. This increase in gas holdup is due to a higher concentration of fine bubbles present throughout the dispersion in FT–300 wax. Bubble sizes associated with FT–300 wax and SASOL reactor wax will be discussed in detail in Chapter V.

In the large diameter column, the foaming capacity of FT–300 wax is greatly reduced. This is primarily due to the increase in liquid mixing (or turbulence) with increasing column diameter (Kato et al., 1972; Heijnen and Van't Riet, 1984). This increase in liquid mixing hinders the production of a stable foam layer at the top of the dispersion and as a result, the nonfoamy or churn–turbulent regime dominates.

For experiments conducted in the batch mode of operation, gas holdups with FT–300 wax are significantly greater than those with SASOL wax at low gas velocities (see Figure 2.32a). In the fully developed churn–turbulent regime (i.e. at $u_g \geq 0.08$ m/s) gas holdups with FT–300 wax and SASOL wax are similar. The same trend was observed in experiments conducted in the batch mode of operation with 20 – 44 μm iron oxide particles (see Figure 2.32b). At a superficial gas velocity of 0.04 m/s the gas holdup with FT–300 was significantly greater than that of SASOL wax (28% for FT–300 and

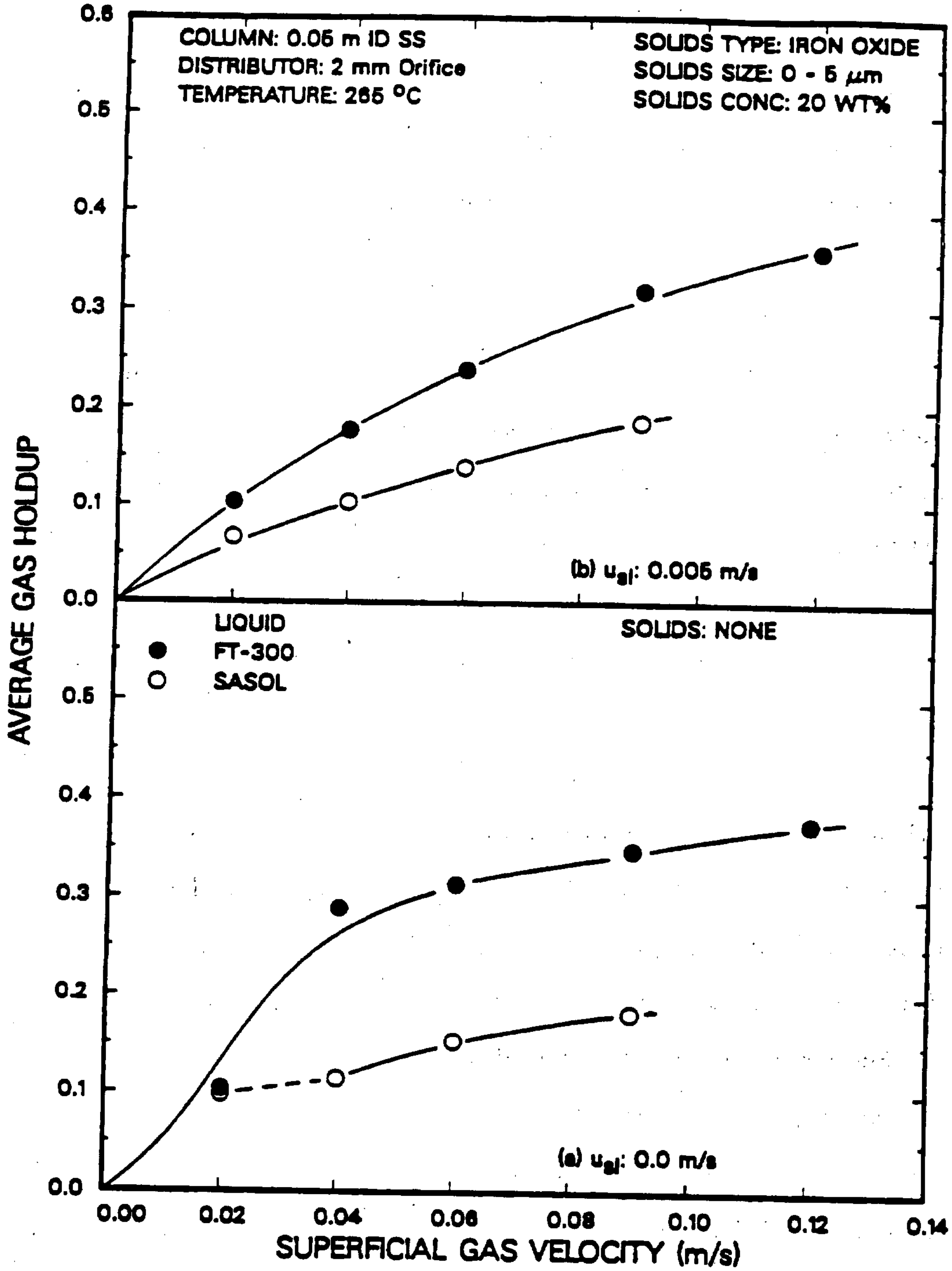

Figure 2.31. Effect of liquid medium on average gas holdup in the 0.05 m ID Column.

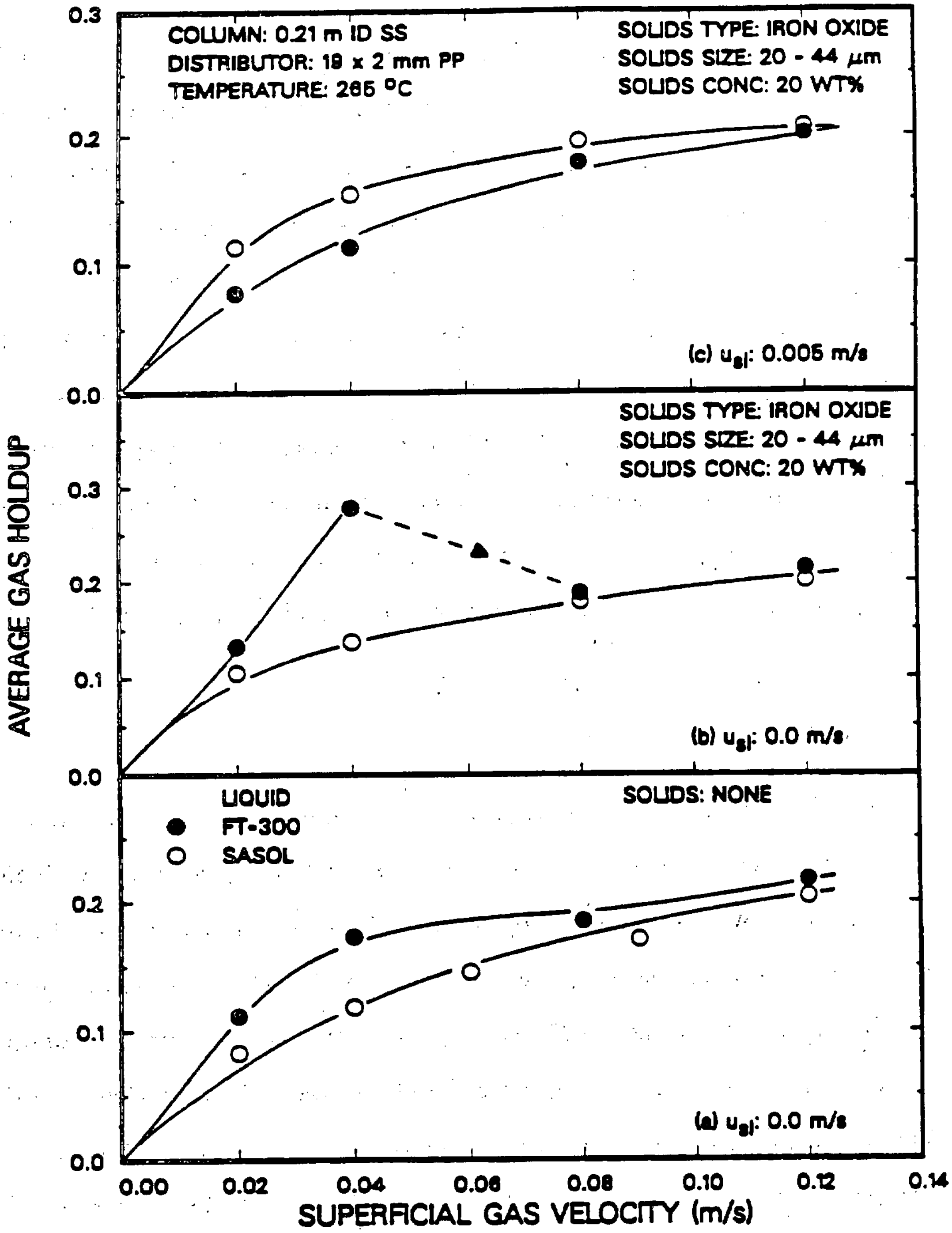

Figure 2.32. Effect of liquid medium on average gas holdup in th 0.21 m ID Column.

14% for SASOL). At higher gas velocities (i.e. $u_g = 0.08$ and 0.12 m/s) the foam layer collapses and gas holdups with FT–300 wax and SASOL wax approach the same value. Gas holdup values obtained with SASOL wax were greater than those with FT–300 wax for experiments conducted in the continuous mode of operation in the presence of large iron oxide particles (Figure 2.32c). As described previously, iron oxide particles appear to be partially nonwettable in SASOL wax, and as a result, when the slip velocity between the gas and liquid phases is reduced, the gas holdup increases. However, at sufficiently high gas velocities, the turbulence created in the large diameter column is sufficient to reduce the adhesion of solid particles to the surface of the tiny gas bubbles, which results in slightly lower holdups. Thus, at higher gas velocities ($u_g \geq 0.08$ m/s), holdup values obtained from the experiments with SASOL reactor wax and FT–300 wax approach the same value.

Effect of Distributor Type

A limited number of experiments were conducted with the bubble cap distributor in the large diameter column. Gas holdup values from experiments with the bubble cap distributor were consistently higher than those from experiments with the 19 x 2 mm perforated plate distributor. Figures 2.33a and 2.33b show results obtained with SASOL ($u_{s\ell} = 0$ m/s) and FT–300 wax ($u_{s\ell} = 0.005$ m/s), respectively. For both waxes, holdups associated with the bubble cap distributor were slightly higher than those with the perforated plate distributor. The jet velocity through both distributors is essentially the same (e.g. at $u_g = 0.12$ m/s, the jet velocity through the perforated plate is 69 m/s while, with the bubble cap it is 63 m/s). Based solely on jet velocities, one would expect the gas holdups to be essentially the same for both distributors. However, we believe that the way in which the gas flows through the two different distributors is the primary cause of the increase in holdup observed with the bubble cap distributor. The

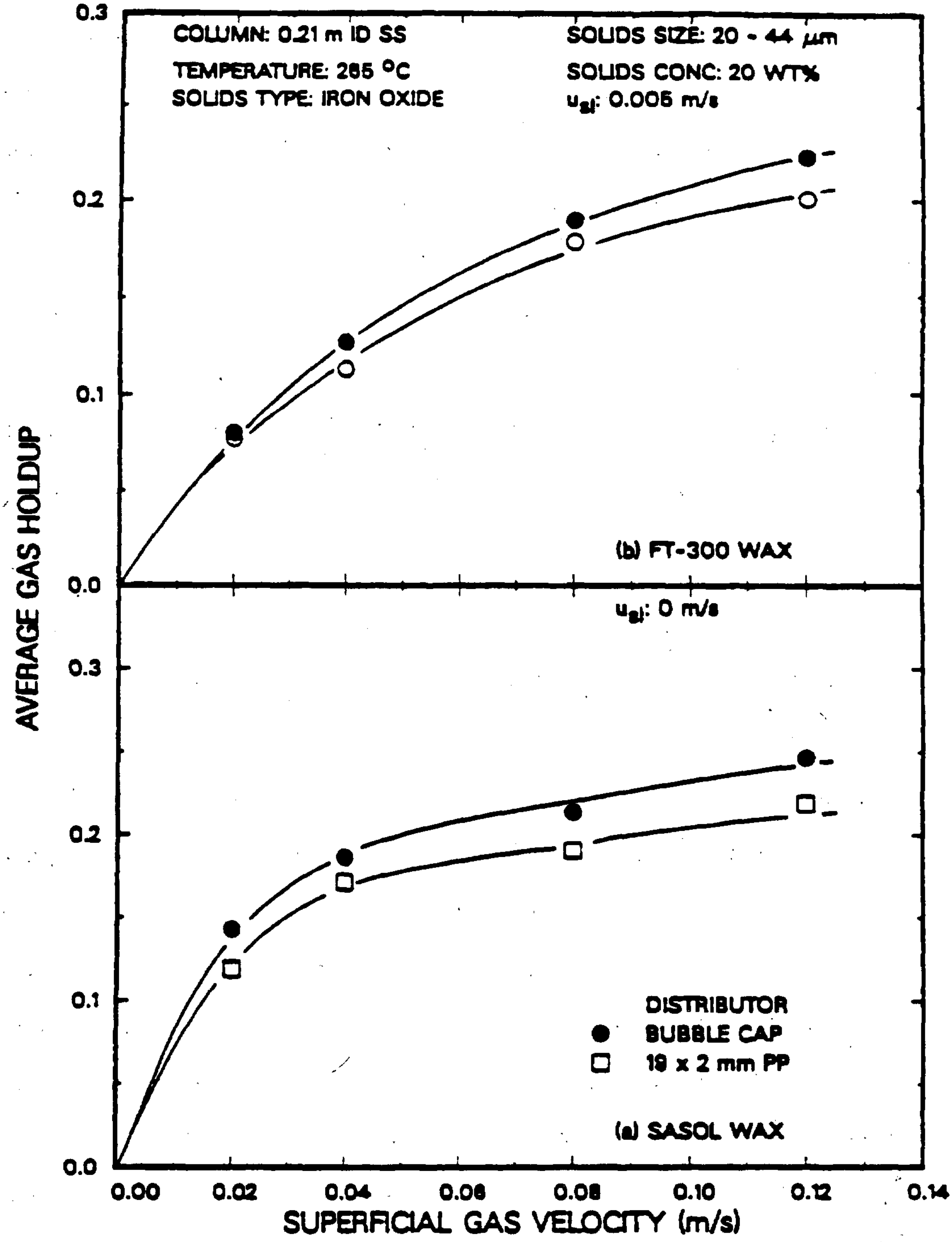

Figure 2.33. Effect of superficial gas velocity and distributor type on average gas holdup (20 – 44 μm iron oxide; (a) SASOL wax, $u_{s\ell} = 0.0$ m/s; (b) FT–300 wax, $u_{s\ell} = 0.005$ m/s).

bubble cap distributor is comprised of seven bubble caps, each with three 2 mm orifices. The flow of gas from each orifice is directed downward towards the distributor (see Figure 2.8). Thus, as the gas bubbles or gas jet exits the openings in the bubble caps, they are broken up by colliding with the distributor plate. On the other hand, as the gas exits the openings in the perforated plate distributor, it flows freely upward through the column; there are no obstacles in its path which may cause bubble breakup. Similar results were obtained for experiments conducted with small iron oxide particles in the batch and continuous modes of operation (see Figures 2.34a and 2.34b, respectively).

Effect of Column Diameter

Gas holdup values from experiments in the small diameter column with FT–300 wax were consistently higher than gas holdup values from experiments in the large diameter column (see Figure 2.35a). The main difference in gas holdups obtained in the two columns is that foam is produced more readily in the small diameter column and once produced, persists over a wider range of gas velocities. During one of the experiments in the small diameter column, the foam broke at a gas velocity of 0.09 m/s (see dashed line in Figure 2.35a) and the gas holdup value was similar to that obtained in the large diameter column. At a gas velocity of 0.02 m/s gas holdups in both columns are similar. This is expected, since at a velocity of 0.02 m/s, the homogeneous bubbling regime exists in both columns. Gas holdups from experiments conducted in the continuous mode of operation in the small diameter column (see Figures 2.35b and 2.35c) were higher than those observed in the large diameter column. It should be pointed out that experiments in the small diameter column were conducted in an increasing order of gas velocities; whereas, experiments in the large diameter column were conducted in a decreasing order of gas velocities. From our previous studies with FT-300 wax (Bukur et al. 1987a,b; Bukur and Daly,1987) it is known that the use of increasing gas

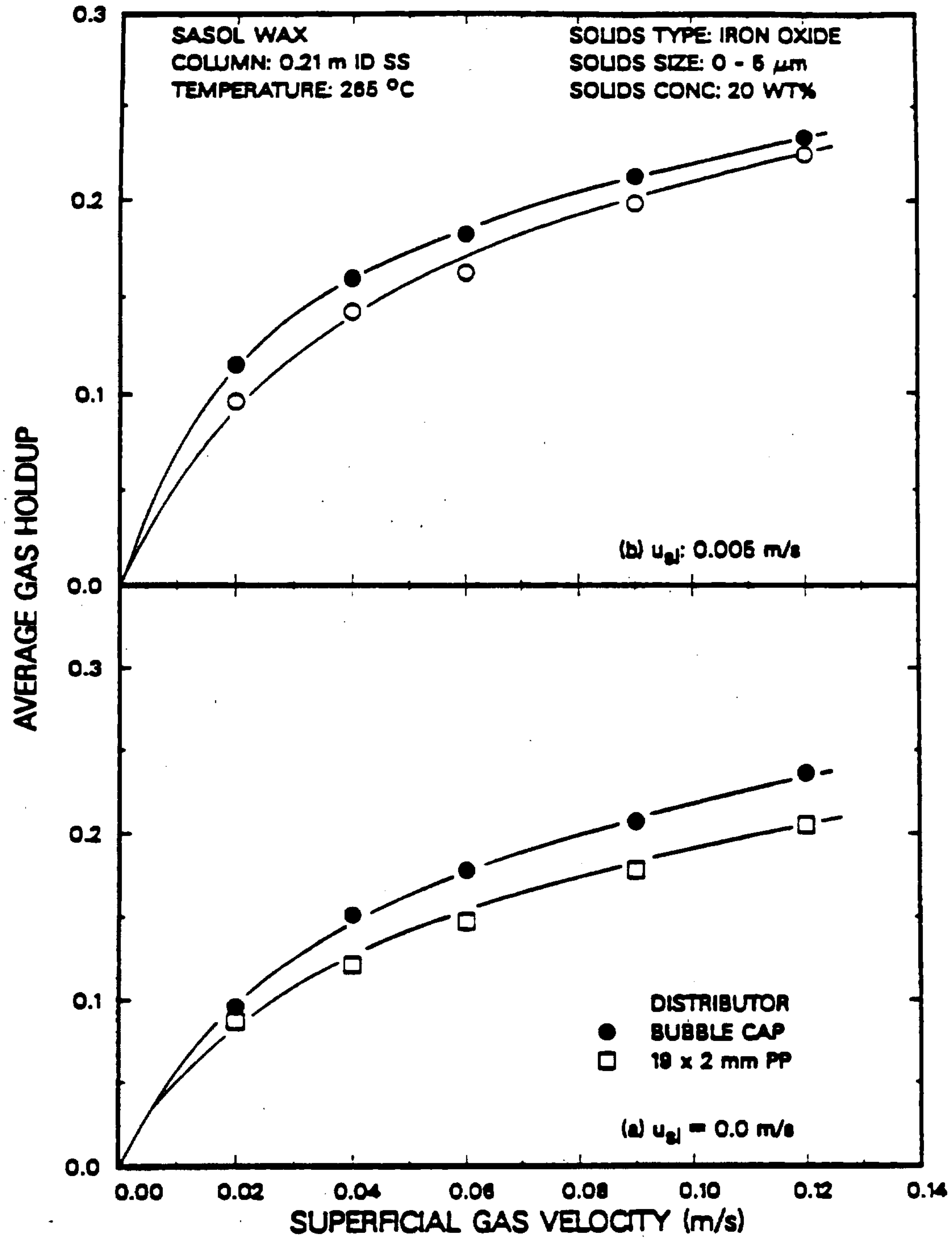

Figure 2.34. Effect of superficial gas velocity and distributor type on average gas holdup with SASOL wax (0 – 5 μm iron oxide; (a) $u_{s\ell} = 0.0$ m/s; (b) $u_{s\ell} = 0.005$ m/s).

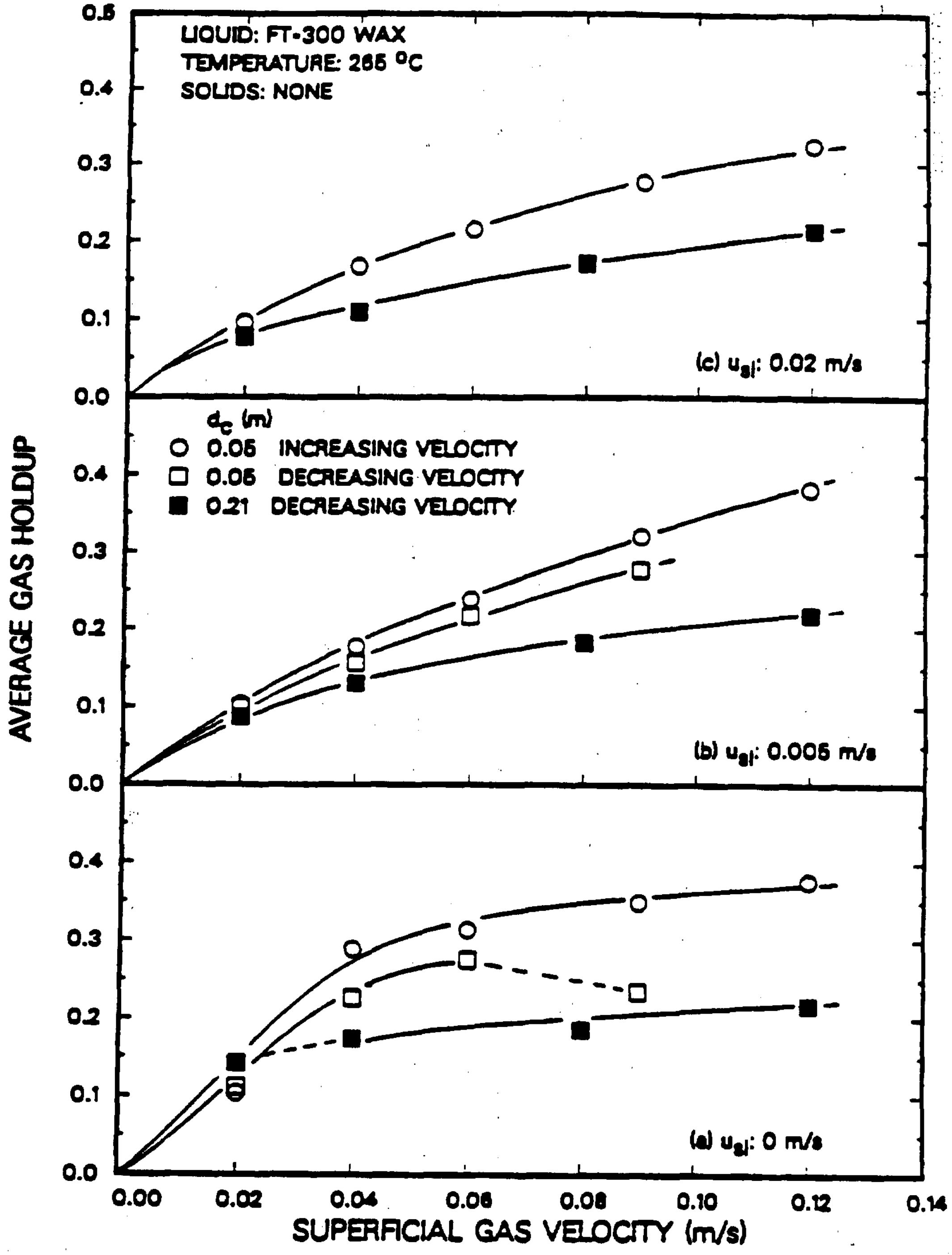

Figure 2.35. Effect of column diameter on average gas holdup with FT–300 wax.

velocities favors higher gas holdups. Thus results shown in Figure 2.35 do not illustrat the effect of column diameter onlybut the effect of operating procedure (increasing vs. decreasing order of gas velocities). The higher holdups observed in the small diameter column may be due to the use of increasing order of gas velocities.

SASOL wax, on the other hand, does not produce foam and gas hodups are not influenced markedly by the order of gas velocities employed. As a result, gas holdups obtained in the 0.05 and 0.21 m ID columns are similar regardless of operating procedures employed (see Figures 2.36a and 2.36b).

Very few experimental studies on the effect of column diameter have been conducted with molten waxes as the liquid medium. Only Mobil workers (Kuo, 1985) and Deckwer et al. (1980) have studied the effect of column diameter with molten waxes. Researchers at Mobil conducted experiments with FT–200 wax (MW = 640) in 0.03 and 0.05 m ID columns, each 2.2 m in height. Their results indicate that for similar jet velocities, column diameter did not have an effect on gas holdup. They also conducted simila studies in two tall columns (0.05 m ID and 0.10 m ID, 9.1 m tall) with FT–200 wax and reactor waxes produced in their bench scale bubble column slurry reactor. These studies showed no effect of column diameter of gas holdup for FT–200 wax; however, with experiments conducted with reactor waxes, slightly higher holdups were obtained in the 0.10 m ID column. Deckwer et al. conducted experiments in two different diameter columns (0.04 m and 0.10 m ID). For temperatures below 250 °C holdups in the smaller diameter column were consistently higher than holdups in the large diameter column for the range of velocities studied (0.005 – 0.03 m/s). Foam was present under these conditions. However, for experiments conducted at temperatures greater than 250 °C, holdup values from the two columns were similar. Reilly et al., 1986 summarized the findings of various researchers for holdups in different diameter columns. They reported that some discrepancy exists as to the effect of column diameter; however, they

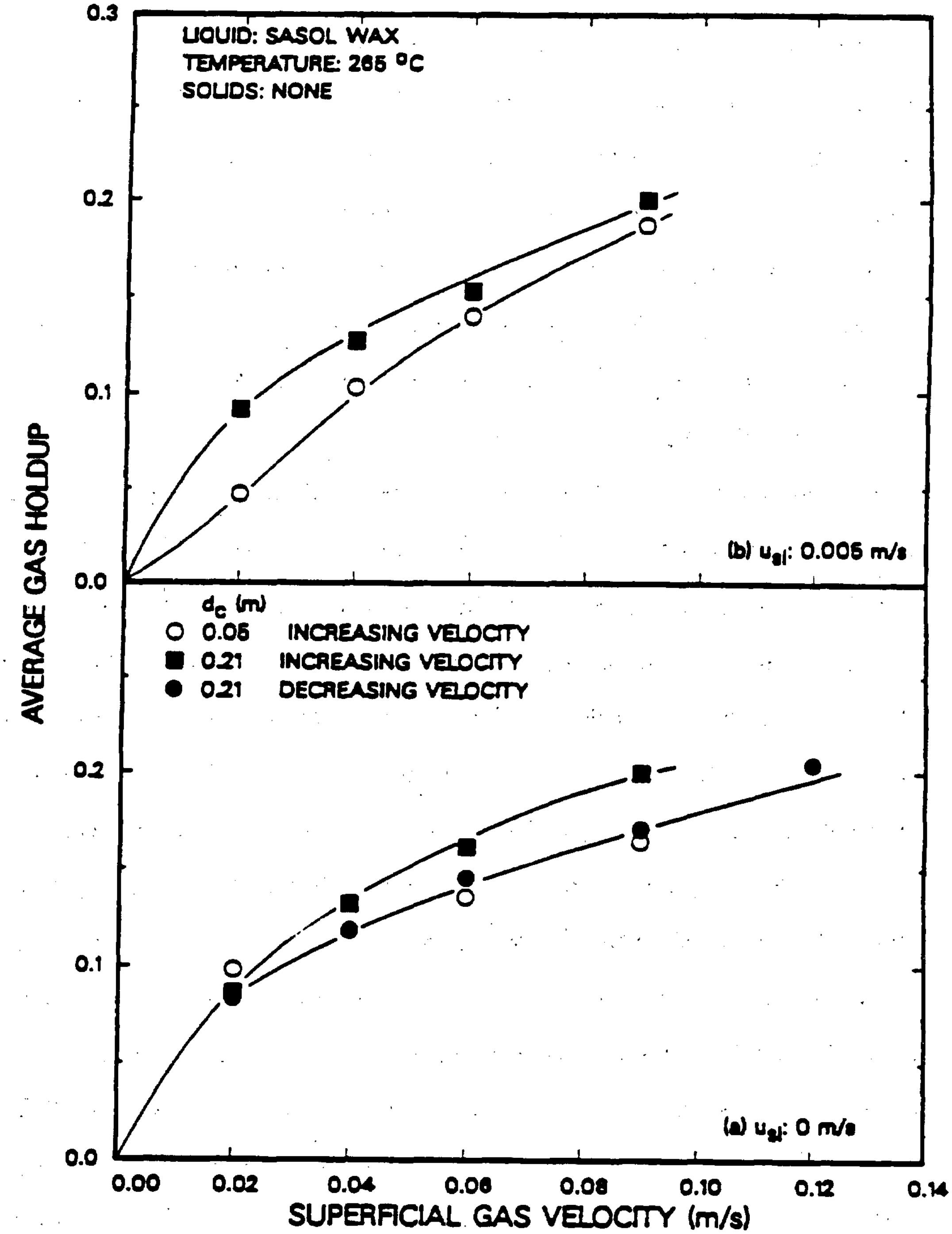

Figure 2.36. Effect of column diameter on average gas holdup with SASOL wax.

pointed out that for columns with diameters greater than 0.10 m, there is essentially no effect of column diameter. Shah et al. (1982) also summarized the findings of various researchers, from holdup measurements made in systems which did not produce foam (mostly air–water), which show that the effect of column diameter on the average gas holdup is minimal. In general, slightly lower holdups were observed in larger diameter columns compared to smaller diameter columns.

Physical Properties and Average Gas Holdup Correlations

Physical Property Measurements

The density and surface tension of FT–300 and SASOL wax were measured at different temperatures. The viscosity of both FT–300 wax and SASOL reactor wax was measured at 230 °C. The physical properties used in developing gas holdup correlations are presented in Table 2.6. The densities of iron oxide and silica particles are 5100 and 2650 kg/m^3, respectively.

Density Measurements

Densities of FT–300 and SASOL wax were measured using the pressure drop across known heights of liquid in the 0.05 m ID glass column. A differential pressure transducer was connected to the bottom of the glass column to measure the pressure drop across the column. The pressure transducer was calibrated with distilled water using the same procedure outlined earlier. The column was filled with the test liquid to a height of 2.5 m and brought to the desired temperature. Once at temperature, the pressure drop was recorded. A portion of the liquid was drained ($\approx$ 0.25 m) and the pressure drop was recorded again. This procedure was repeated until the liquid level in the column was approximately 1 m. The density of the wax at a given temperature was obtained from the slope of the pressure drop versus height plot, after appropriate corrections for the calibration factor.

Table 2.6. Physical Properties of FT-300 Wax and SASOL Wax

LIQUID	TEMPERATURE (°C)	DENSITY (kg/m^3)	VISCOSITY[a] FRESH WAX (kg/m-s)	VISCOSITY USED WAX (kg/m-s)	SURFACE TENSION FRESH WAX (N/m)	SURFACE TENSION[b] USED WAX (N/m)
FT-300	150	–	0.0064		0.024±0.0004	0.025
	200	722	0.0042		0.021±0.0006	0.02±0.001
	230	706	0.0036 (0.0023)[c]	0.0026–0.0041[d]	0.019±0.0005	0.019
	260	681	0.0028		0.017±0.001	0.017±0.0005
SASOL	150	–	0.0042		0.024	0.019±0.0005
	200	701	0.003		0.02±0.001	0.017±0.001
	230		0.0025		0.019	0.016±0.0009
	260	655	0.0022		0.016±0.0003	0.014±0.0008

[a] From Bukur et al., (1987c)

[b] Based on analysis of several samples – all contained solids

[c] Single measurement during this project

[d] Range of values (lowest for sample with no solids; highest for sample taken from slurry containing 30 wt% silica)

Viscosity Measurements

Viscosity measurements were made in a Brookfield viscometer (LV series, 2.5X) using a cylindrical spindle (SC4-18) operating at 60 RPM. A Brookfield Thermosel system allowed measurements up to temperatures of 250 °C. The system was first calibrated using fluids of known viscosities. Three fluids were used; water (0.01 kg/m-s), and two viscosity standards (.051 and .081 kg/m-s – supplied by Brookfield). The standards were used before and after viscosity measurements with wax to monitor errors due to device drift. Each measurement required an 8 ml sample of the test fluid.

Results from these measurements together with those presented by Bukur et al. (1987c) are presented in Table 2.6. The viscosity of the fresh FT–300 sample at 230 °C obtained in the current study was significantly lower than that previously obtained (i.e. 0.0023 kg/m–s vs 0.0035 kg/m–s). The reason for this discrepancy is not known. Several samples of used wax were also analyzed, one without any solids (0.0026 kg/m–s) and several samples from experiments conducted with solids (both iron oxide and silica). The samples from the experiments conducted with solids were prepared as follows. The solidified slurry sample was melted and the solids were allowed to settle. The liquid was decanted and the viscosity of the decanted liquid was measured. The viscosity of wax from experiments with solids was higher than that from experiments without solids. More than likely, the observed increase in viscosity was due to the presence of some solids in the samples. The viscosity was highest (0.0041 kg/m–s) for the sample from the experiment conducted with 30 wt% 0 – 5 μm silica particles.

Surface Tension Measurements

Surface tension measurements were made using a Fischer Model 215 Autotensiomat. The surface tension apparatus was modified for high temperature measurements, as suggested by the manufacturer. The surface tension was measured three times for each sample at a given temperature using both fresh and used FT–300 and SASOL wax. The average surface tension values from these measurements are given in Table 2.6. Some of the surface tension values presented in Table 2.6 are average values based on analysis of more than one sample. For these values, the standard deviation is also given.

Jasper (1972) presents surface tension data for normal paraffins (C5 – C20, C26 and C60). The values reported by Jasper for C5 – C20 paraffins were obtained at temperatures between 10 and 120 °C and for C26 and C60 paraffins, surface tension values were obtained for temperatures up to 180 °C. According to Jasper, surface tension is a linear function of temperature for reduced temperatures ($T/T_{critical}$) of 0.4 to 0.7. Thus, for the data he presented, he also gave values of the slopes and intercepts obtained from a plot of surface tension versus temperature. Figure 2.37 shows the effect of temperature on surface tension for data obtained in this study. Surface tension values for fresh FT–300 wax, used FT–300 wax, and fresh SASOL wax are similar and they vary linearly with temperature. The surface tensions of used SASOL wax were consistently lower than those of fresh SASOL wax (Table 2.6 and Figure 2.37).

The surface tension values for fresh FT–300 and fresh SASOL wax were fitted to the following equation using linear regression

$$\sigma = \text{int} - \text{slope} * T \tag{2.29}$$

where σ is the surface tension in dynes/cm and T is the temperature in °C. The following slopes and intercepts were obtained

FRESH FT–300: SLOPE = 0.0606, INT = 33.1

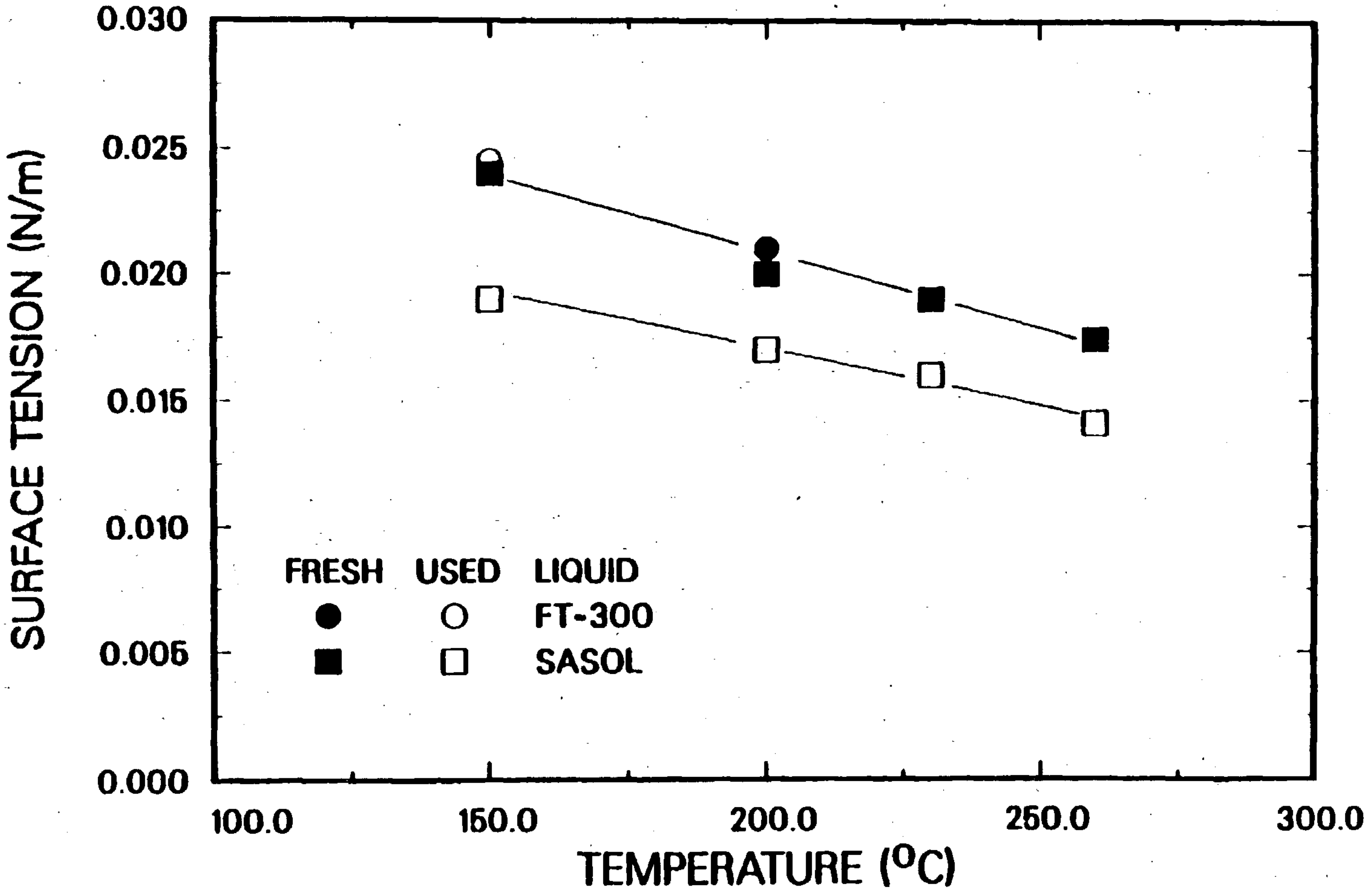

Figure 2.37. Effect of temperature on surface tension of fresh and used FT-300 and SASOL wax.

FRESH SASOL: SLOPE = 0.0659, INT = 33.7

FT–300 wax has a molecular weight of 730, which corresponds to a carbon number of 52. For a C26 paraffin, the slope and intercept values reported by Jasper were 0.07332 and 31.46, respectively and for a C60 paraffin the slope and intercept values were 0.05827 and 30.89, respectively. The results (i.e. slopes and intercepts) obtained in this study are in good agreement with the values reported by Jasper.

Gas Holdup Correlations

Numerous empirical correlations for predicting gas holdup in nonfoaming systems have been published (e.g. Hughmark, 1967; Akita and Yoshida, 1974; Bach and Pilhofer, 1978; Kara et al., 1982; Hatate et al., 1986, Badjugar et al., 1986, Zheng et al., 1988). The correlations evaluated in this study are presented in Table 2.7. Some researchers (e.g. Smith and Ruether, 1985; Fan, 1989) have found that in systems with low solids concentrations, correlations developed for two–phase systems can be applied to three–phase systems, if the physical properties (i.e density and viscosity) of the liquid are replaced by those of the slurry. The slurry density, $\rho_{s\ell}$ is obtained from:

$$\frac{1}{\rho_{s\ell}} = \frac{1}{\frac{\omega_s}{\rho_s} + \frac{1-\omega_s}{\rho_\ell}} \tag{2.30}$$

and the slurry viscosity may be estimated from

$$\mu_{s\ell} = \mu_\ell \frac{1 + 0.5\epsilon_s}{(1 - \epsilon_s)^4} \tag{2.31}$$

Equation 2.31 is valid for $\epsilon_s < 0.4$ (Perry and Chilton, 1983), where ϵ_s is the volume fraction of solids in the liquid/solid slurry.

Average gas holdup results from our study can be divided into two groups: (1) results in which foam was observed and (2) results in which no foam was observed.

Table 2.7. Summary of Gas Holdup Correlations Presented in the Literature

CORRELATION	COLUMN ID	CONDITIONS	SYSTEM	REFERENCE
$\epsilon_g = a Fr^b Ar^c Bo^d (1 + u_\ell / u_g)^e (1 - \epsilon_s)^f$ $Fr = \frac{u_g^2}{d_c g}$, $Ar = \frac{d_p^3 g(\rho_s - \rho_\ell)}{\rho_\ell \mu_\ell^2}$, $Bo = \frac{g d_p^2 \rho_\ell}{\sigma}$ where a, b, c, d, and e, are adjustable parameters which depend on flow regime	0.285 m	$0 < u_g < 0.16$ $0 \leq u_\ell \leq 0.04$ $\rho_\ell = 1000$ $d_p = 615$	gas: air liquid:water solids:glass spheres	Zheng et al., 1988
$\epsilon_g = 0.9(1 - \omega_s)^{0.7} u_g^{0.525}$	0.076 and 0.301 m	$0 < u_g \leq 0.14$ $0 \leq \omega_s \leq 0.20$ $751 \leq \rho_\ell \leq 1000$ $0.001 \leq \mu_\ell \leq 0.0013$ $49 \leq d_p \leq 107$	gas: air liquid: water, soltrol solids: glass spheres, ammonia synth. cat., Triple A cat., FCC	Badgujar et al., 1986
$\epsilon_g = \frac{u_g/(u_g + u_\ell)}{1.20 + 0.35/\sqrt{Fr_\ell}}$ $Fr_\ell = \frac{(u_g + u_\ell)^2}{g d_c}$	1.55 and 2.6 m	$0 \leq u_g < 5$ $u_\ell = 0.15$ and 0.6 $0 \leq \omega_s \leq 0.60$ $\rho_\ell = 1000$ $d_p = 30, 63,$ and 100	gas: air liquid: tap water solids: glass spheres	Hatate et al., 1985

Table 2.7. (cont.)

CORRELATION	COLUMN ID	CONDITIONS	SYSTEM	REFERENCE
$\epsilon_g = \dfrac{Re_g}{A + BRe_g + CRe_{s\ell} + D\left(\frac{\epsilon_s}{\epsilon_s + \epsilon_\ell}\right)}$	0.152 m	$0 < u_g \leq 0.3$	gas: air	Kara et al., 1982
$Re_{s\ell} = \dfrac{d_c u_{s\ell} \rho_{s\ell}}{\mu_{s\ell}}$, $Re_g = \dfrac{d_c u_g \rho_g}{\mu_g}$		$0 \leq u_{s\ell} \leq 0.1$	liquid: water	
where A, B, C, D, and E are adjustable parameters which depend on particle size		$0 \leq \frac{c_s}{\rho_{s\ell}} \leq 0.4$ $d_p = 0, 10, 30, 70$	solids: coal, dried mineral	
$\epsilon_g = (2 + (0.35 / u_g)[(\rho_\ell / 1000)(\sigma_\ell / 0.072)]^{0.33})^{-1}$	< 0.1 m	$0.004 \leq u_g \leq 0.45$	gas: air	Hughmark, 1967
		$780 \leq \rho_\ell \leq 1700$	liquid: water, kerosene	
		$0.0009 \leq \mu_\ell \leq 0.152$	glycerol aqu. soln.,	
		$0.025 \leq \sigma \leq 0.076$	light oil, salt solns.	

u_i (m / s), ρ_i (kg / m^3), μ_i (kg / m – sec) where $i = g, s\ell, \ell$

d_p (μm), σ_ℓ(N / m)

Deckwer et al. (1980) used the following empirical correlation to correlate hol values obtained using molten paraffin wax in the foamy regime

$$\epsilon_g = 8.4u_g^{1.1} \pm 0.015 \qquad u_g \leq 0.04 \mathrm{m/s} \tag{2.32}$$

Researchers at Mobil (Kuo, 1985) used a similar correlation to predict gas holdups with FT–200 wax (MW=630) under foaming conditions.

$$\epsilon_g = 10.3u_g^{1.1} \qquad u_g \leq 0.06 \mathrm{m/s} \tag{2.33}$$

The correlations presented above were obtained from experiments conducted in the batch mode of operation. While the two correlations are similar, the difference in the constant (8.4 and 10.3) is probably due to differences in the foaming characteristics of the systems studied. Two correlations were developed by Bukur et al. (1987a) for data obtained in the foamy regime. One correlation was developed from gas holdup data obtained using orifice plate distributors

$$\epsilon_g = 0.94u_g^{0.41} \qquad 0.01 \leq u_g \leq 0.07 \mathrm{m/s} \tag{2.34}$$

and the other correlation was developed from gas holdup data obtained using a 40 μm sintered metal plate distributor

$$\epsilon_g = 1.06u_g^{0.15} \qquad 0.01 \leq u_g \leq 0.12 \mathrm{m/s} \tag{2.35}$$

The correlations proposed by Deckwer et al. and Kuo (Eqs. 2.32 and 2.33, respectively) show that holdup increases almost proportionally with superficial gas velocity, while results from the study by Bukur et al. show that holdup values tend to level off at higher gas velocities. A possible explanation for the discrepancy is the range of gas velocities employed in the three studies. The studies by Deckwer et al. and Kuo were limited to low gas velocities, where the holdup increases linearly with gas velocity.

However, at higher gas velocities, holdup values level off. Thus, it is evident, that a single correlation cannot be developed for predicting holdup values in the foamy regime. Therefore, the correlations developed in the present study are based on data obtained in the slug flow and churn–turbulent flow regimes. In particular, gas holdup data obtained in the 0.05 m ID stainless steel column in the batch mode of operation with FT–300 wax have been omitted.

Measured gas holdups values were compared with values predicted using the correlations presented in Table 2.7. The correlations developed by Hughmark (1967) and Hatate et al. (1986) were based on data obtained from two–phase systems. For Hughmark's correlation, the slurry density was used as opposed to the density of the liquid. Since constants in Zheng et al.'s correlation depend on the flow regime, it was assumed that at a gas velocity of 0.02 m/s the homogeneous bubbling regime prevails, at a gas velocity of 0.04 m/s the transition regime exists, and for gas velocities greater than 0.04 m/s the column was assumed to operate in either the churn–turbulent (0.21 m ID) or slug flow (0.05 m ID) regime. The correlation presented by Kara et al. (1982) has variable parameters as well. The constants change depending on the size of particles used. Thus, in applying Kara et al.'s correlation to our system, we used constants for 10 μm particles to estimate gas holdups for slurries containing 0 – 5 μm particles, and constants for 30 μm particles to estimate gas holdups for slurries containing 20 – 44 μm particles. The number of data points associated with a given set of conditions, which were used in the correlations are presented in Table 2.8. A total of 222 points were used. Mean square errors (MSE), defined as

$$\mathrm{MSE} = \frac{\sum_i (\epsilon_{\mathrm{meas}_i} - \epsilon_{\mathrm{pred}_i})^2}{n-1} \qquad i = 1 \text{ to } n \tag{2.36}$$

were first estimated using the original values of constants in the literature correlations. The MSE values were between 0.0015 to 0.017 (Table 2.9). The magnitude of the

Table 2.8. Summary of Number of Points at a Given Set of Conditions

LIQUID	COLUMN ID (m)	SOLIDS TYPE	SOLIDS SIZE μm	$u_{s\ell}$ (m/s)	No. Pts.
FT–300	0.05	None	–	0.005	4
FT–300	0.05	Iron oxide	0 – 5	0.005	12
FT–300	0.05	Iron oxide	0 – 5	0.02	12
FT–300	0.05	Silica	0 – 5	0.005	12
FT–300	0.05	Silica	0 – 5	0.02	4
FT–300	0.05	Iron oxide	20 – 44	0.005	7
FT–300	0.05	Iron oxide	20 – 44	0.02	3
FT–300	0.05	Silica	20 – 44	0	4
FT–300	0.21	None	–	0	4
FT–300	0.21	None	–	0.005	4
FT–300	0.21	None	–	0.02	4
FT–300	0.21	Iron oxide	20 – 44	0	3
FT–300	0.21	Iron oxide	20 – 44	0.005	8
SASOL	0.05	None	–	0	8
SASOL	0.05	Iron oxide	0 – 5	0.005	4
SASOL	0.05	Iron oxide	20 – 44	0	4
SASOL	0.21	None	–	0	9
SASOL	0.21	None	–	0.005	12
SASOL	0.21	None	–	0.02	4
SASOL	0.21	Iron oxide	0 – 5	0	24
SASOL	0.21	Iron oxide	0 – 5	0.005	15
SASOL	0.21	Iron oxide	0 – 5	0.02	5
SASOL	0.21	Iron oxide	20 – 44	0	16
SASOL	0.21	Iron oxide	20 – 44	0.005	12
SASOL	0.21	Iron oxide	20 – 44	0.02	12
SASOL	0.21	Silica	20 – 44	0	4
SASOL	0.21	Silica	20 – 44	0.005	8
SASOL	0.21	Silica	20 – 44	0.02	4

MSE is a measure of the goodness of fit, and a smaller value implies better agreement between the measured and predicted values. We then calculated new values of constants in these correlations by minimizing the MSE via non-linear regression (NLIN procedure in SAS). The MSE values obtained using new values for the constants in the existing correlations were slightly smaller than those obtained when the original constants were employed as shown in Table 2.9.

Figure 2.38 compares parity plots obtained using the original correlations proposed by Badjugar et al. (1986) and Hughmark (1967) (Figures 2.38a and 2.38c, respectively) with those for the same two correlations after the constants were recalculated (Figures 2.38b and 2.38d, respectively). The correlation proposed by Badjugar et al. is a three–phase correlation, and the correlation proposed by Hughmark is a two–phase correlation. For Hughmark's correlation, the liquid density was replaced by the slurry density (see Eq. 2.30). In Figures 2.38a and 2.38b (Badjugar et al. correlation) 85% of the measured gas holdup values are within ±30% of the predicted values using the original constants (Figure 2.38a) and 94 % of the measured holdup values were with ±30% of the predicted values using the new constants(Figure 2.38b). Similar results (i.e. better agreement between predicted and measured holdup values) were obtained with Hughmark's correlation (see Figures 2.38c and 2.38d). It is also evident from Figure 2.38 that a two–phase correlation may be used to predict gas holdups in three–phase Fischer–Tropsch slurry bubble columns in the slug flow and churn–turbulent flow regimes.

The lowest MSE (0.0007) was obtained using Zheng et al.'s correlation with recalculated constants (see Table 2.9). This was expected since this correlation has the largest number of adjustable parameters. However, the difference in mean square errors between Zheng et al.'s correlation and Badjugar et al's correlation with recalculated constants was not that significant (0.0007 vs. 0.0010) even though there are twice as

2.9. Mean Square Errors for Literature Correlations

CORRELATION[a]	MSE[b]	MSE[c]	REFERENCE
$\epsilon_g = 0.46Fr^{0.26}Ar^{-0.009}Bo^{0.06}(1 + u_{s\ell} / u_g)^{0.04}(1 - \epsilon_s)^{0.19}$	0.0018	0.0007	Zheng et al., 1988[d]
$\epsilon_g = 0.7(1 - \omega_s)^{-0.08}u_g^{0.5}$	0.0015	0.0010	Badgujar et al., 1986
$\epsilon_g = \frac{u_g/(u_g+u_\ell)}{2.75+0.12/\sqrt{Fr_\ell}}$	0.0017	0.0014	Hatate et al., 1986
$\epsilon_g = \frac{Re_g}{103.7+4.65Re_g+0.19Re_{s\ell}-573\left(\frac{\epsilon_s}{\epsilon_s+\epsilon_\ell}\right)}$	0.0045	0.0032	Kara et al., 1982
$\epsilon_g = (2.74 + (0.29 / u_g)[(\rho_\ell / 1000)(\sigma_\ell / 0.072)]^{0.33})^{-1}$	0.0015	0.0012	Hughmark, 1967

[a] reevaluated constants

[b] based on original constants; 222 data points

[c] based on reevaluated constants; 222 data points

[d] Bond number is defined with respect to column diameter as opposed to particle diameter

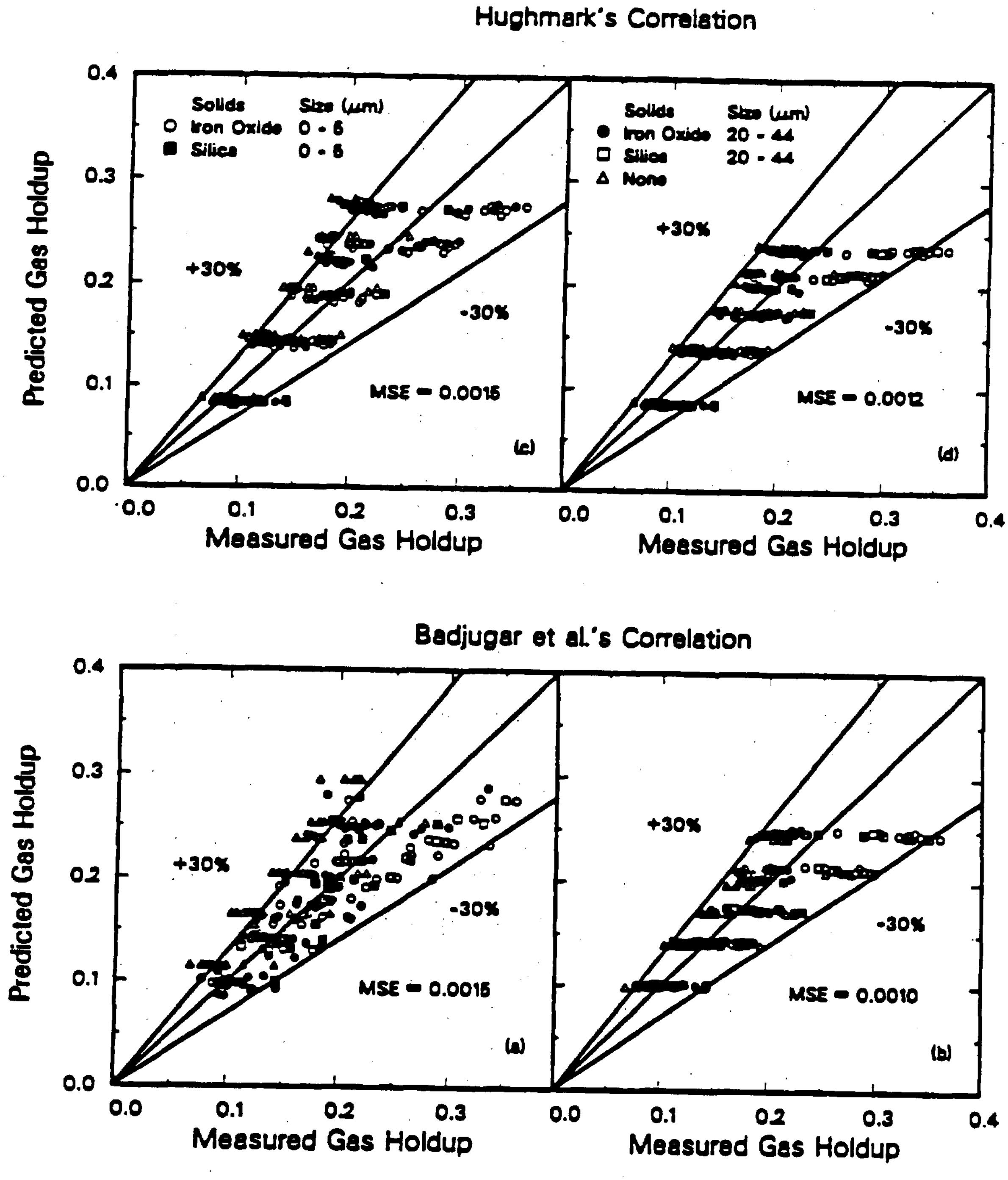

Figure 2.38. Parity plot of predicted versus measured gas holdup
((a and b) Badjugar et al., 1986; (c and d) Hughmark, 1967).

many adjustable parameters in the correlation proposed by Zheng et al. This is not surprising since we did not observe a significant effect of particle size, solids concentration, or slurry flow rate on gas holdup in the slug flow (0.05 m ID column) or churn–turbulent flow (0.21 m ID column) regimes. The following terms in the correlation proposed by Zheng et al., did not vary significantly over the range of conditions employed in this study

$$1.15 < \mathrm{Ar}^{-0.009} < 1.21$$

$$1.0 < (1-\epsilon_s)^{0.19} < 1.028$$

$$0.988 < (1+u_{s\ell}/u_g) < 1.0$$

The correlation proposed by Badjugar et al. with recalculated constants also indicated that there was no significant effect of solids concentration on gas holdup, i.e.

$$1.0 < (1-\omega_s)^{-0.08} < 1.03$$

Neglecting the terms presented above, the correlations proposed by Zheng et al., and Badjugar et al. are similar, with the exception of the fact that the former takes into account column diameter; whereas, the latter does not.

Since there was a negligible effect of solids size and concentration and slurry flow rate, the following dimensionless correlation was selected for further evaluation

$$\epsilon_g = a\mathrm{Fr}_g^b\mathrm{Bo}^c \tag{2.37}$$

We observed an effect of column diameter for gas holdups with FT–300 wax (see Figure 2.35). Eq. 2.35 was evaluated using either all data points (222) or omitting those associated with FT–300 wax in the small stainless steel column (165 points). Table 2.10 summarizes the parameters and MSE's associated with this analysis. Figures 2.39a and 2.39b are parity plots of the measured gas holdup values versus the predicted gas holdup

Table 2.10. Goodness of Fit and Parameters for Empirical Holdup Correlation.

CORRELATION:

$$\epsilon_g = a\ (Fr)^{b}\ (Bo)^{c}$$

$$\text{where:} \quad Fr = \frac{u_g^2}{g\, d_c} \quad ; \quad Bo = \frac{d_c^2 \varrho_{sl}\, g}{\sigma_l}$$

Number of Points	222	165
MSE	.0007	.0004
% Points within 30%	90	95
Parameters:		
a	0.51	0.24
b	0.26	0.22
c	0.05	0.11

Range of Variables:

$0 < u_g < 0.12$ m/s, $u_l = 0, 0.005, 0.02$ m/s , $d_c = 0.05$ and 0.21 m $\sigma_l = 0.016 - 0.017$ N/m, $0 < \epsilon_{sl} < 0.1$, $\varrho_s = 5100$ and 2650 kg/m^3, $\varrho_l = 660$ and 680 kg/m^3 , $d_p = 0$–5 and 20–44 μm, $\mu_{sl} = 0.028$ and 0.022 kg/(m–s)

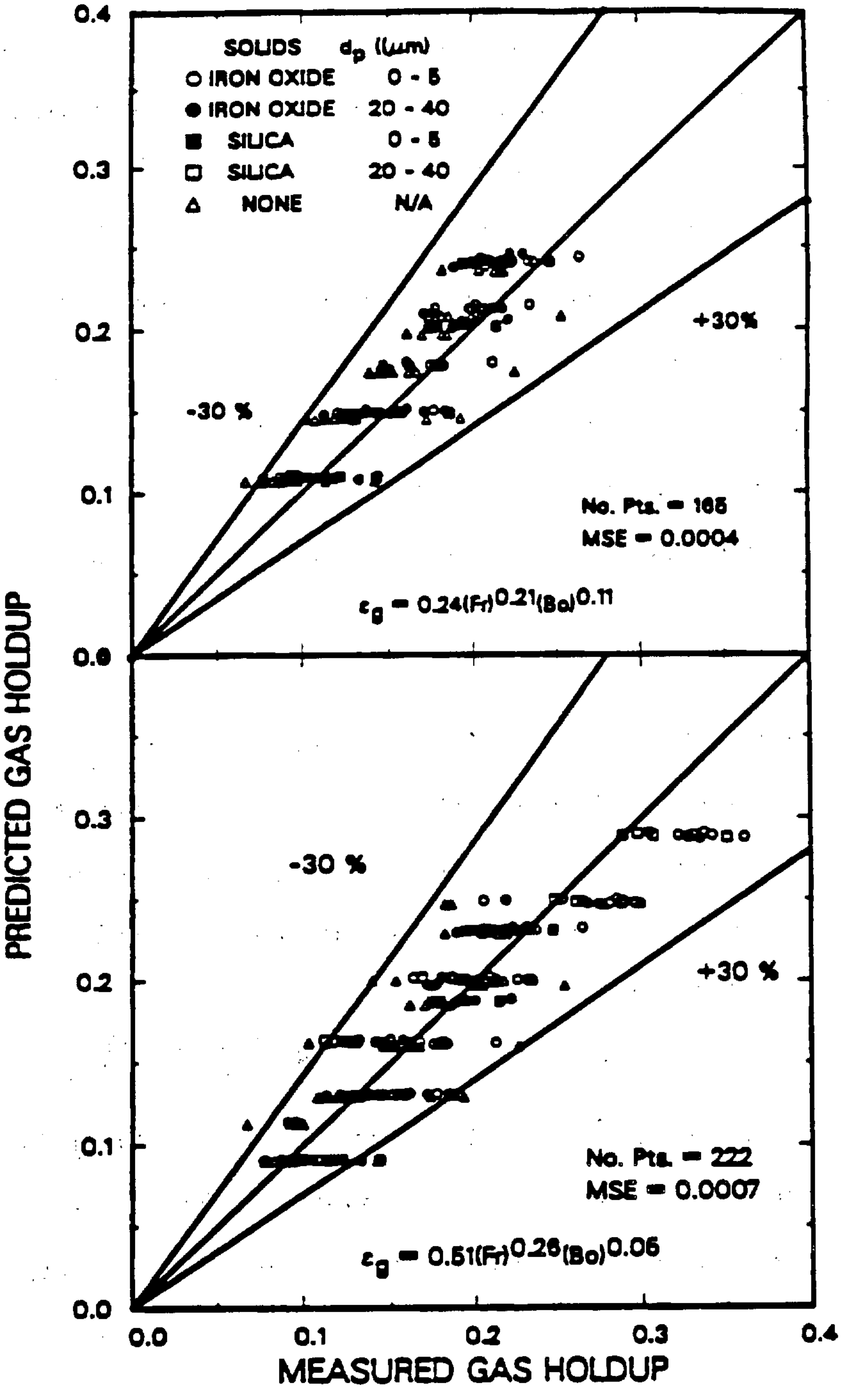

Figure 2.39. Parity plot of predicted versus measured gas holdups for the correlations developed in this study.

values when all data points (i.e. 222) were used and when data for FT–300 wax in the small diameter column were omitted (165 points), respectively. Approximately 90% of the predicted gas holdup values were within ± 30% of the measured values when all data points were used and 95% of the measured gas holdup values were within ±30% when data from the small diameter column for FT–300 wax were excluded.

Extensive two–phase studies were conducted by Bukur et al. (1987a,c) using FT–300 wax, FT–200 wax, Mobil reactor wax, and SASOL reactor wax in the glass columns. An empirical correlation was developed using 349 data points in the slug flow and churn–turbulent regimes. The correlation developed was similar to Eq. 2.37:

$$\epsilon_g = 0.247(Fr_g^{0.30}Bo^{0.15}) \tag{2.38}$$

Data from both our three–phase studies (excluding gas holdups in the small diameter column with FT–300 wax) and two–phase studies (Bukur et al.) were combined and the following general correlation was developed which may be used to predict gas holdups in Fischer–Tropsch slurry bubble column reactors operating in the slug flow or churn–turbulent regime:

$$\epsilon_g = 0.24(Fr_g^{0.28}Bo^{0.14}) \tag{2.39}$$

The MSE based on 514 data points was 0.0007. Figure 2.40 is a parity plot of the measured versus predicted gas holdups using Eq. 2.39. Approximately 94% of the experimental data were within ± 30% of the predicted values.

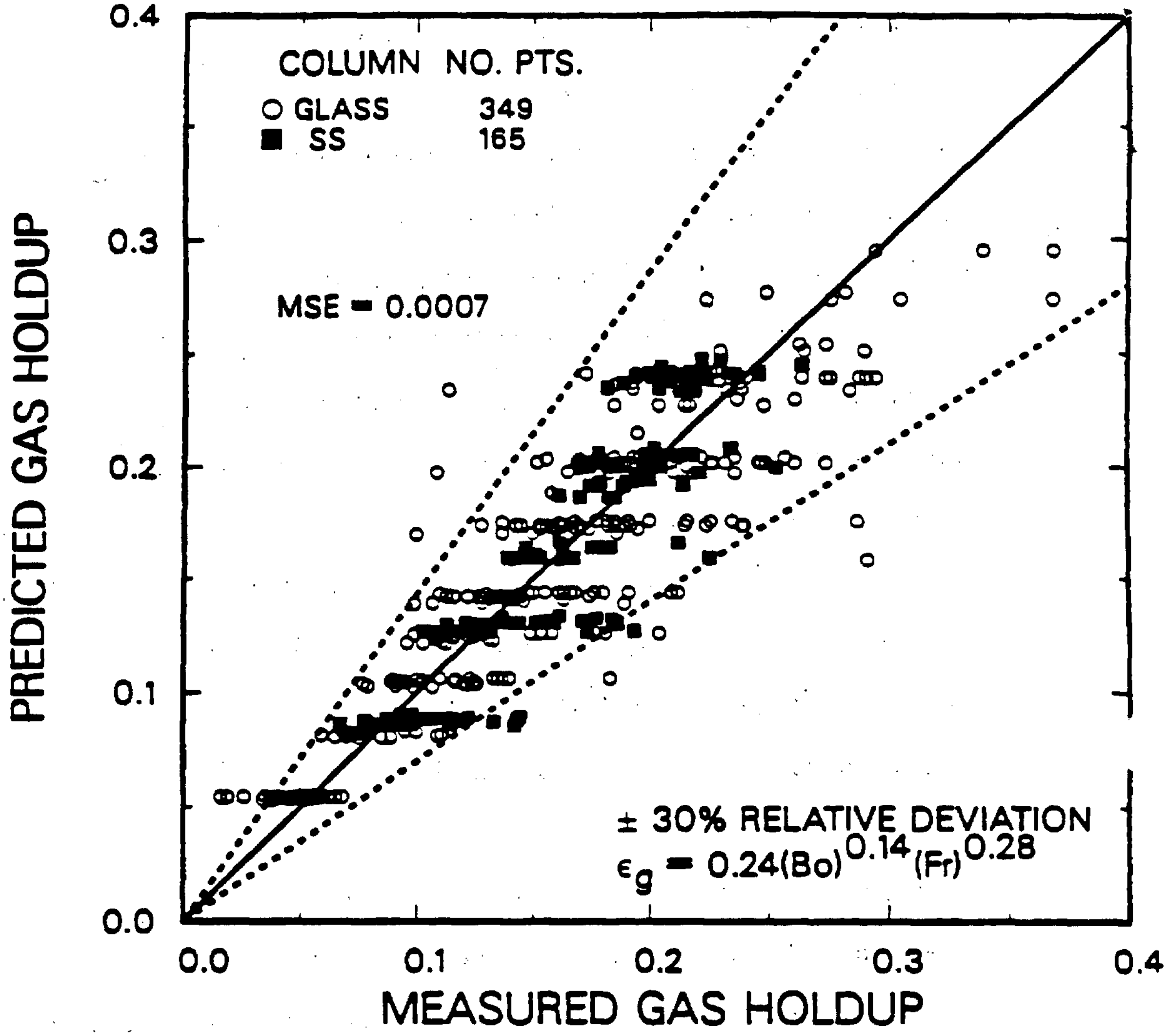

Figure 2.40. Parity plot of predicted versus measured gas holdup (wax type: SASOL, FT-300, Mobil; u_g = 0.01 to 0.15 m/s; $u_{s\ell}$= 0, 0.005, and 0.02 m/s; d_c = 0.05 and 0.21 m ID; solids: 0, 10, 20, and 30 wt% iron oxide and silica).