OAK RIDGE NATIONAL LABORATORY
OPERATED BY
UNION CARBIDE CORPORATION
NUCLEAR DIVISION

POST OFFICE BOX X
OAK RIDGE, TENNESSEE 37830

ORNL/MIT-250

DATE: March 22, 1977 COPY NO.

SUBJECT: Dispersion in a Three-Phase Fluidized Bed

Authors: D.R. Christman, S.C. Loftus, and M.P. Rill

Consultants: J.M. Begovich and J.S. Watson

ABSTRACT

The effects of liquid and gas superficial velocity and solids loading on axial dispersion in a three-phase fluidized bed were investigated. Gas velocities of 1.4 to 9.3 cm/sec and liquid velocities of 4.4 to 13.2 cm/sec were employed with 2 and 3 kg of 0.46-cm-diam, 2.24 gm/cm^3 glass beads. Dispersion coefficients were calculated by analysis of moments and a transfer function method. There was substantial disagreement between the two methods. Dispersion coefficients calculated by the analysis of moments were most dependent on the liquid velocity and ranged from negative values to 200 cm^2/sec.

Oak Ridge Station
School of Chemical Engineering Practice
Massachusetts Institute of Technology
W.M. Ayers, Director

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road, Springfield, Virginia 22161
Price: Printed Copy $4.00; Microfiche $3.00

Contents

		Page
1.	Summary	4
2.	Introduction	4
	2.1 Background	4
	2.2 Objectives and Method of Attack	5
	2.3 Theory	5
3.	Apparatus and Procedure	7
	3.1 Apparatus	7
	3.2 Procedure	7
4.	Results and Discussion	10
	4.1 Liquid Volume Fraction	10
	4.2 Dispersion Coefficients	15
5.	Conclusions	15
6.	Recommendations	19
7.	Acknowledgments	19
8.	Appendix	20
	8.1 Derivation of Transfer Function	20
	8.2 Calculation of Holdups	22
	8.3 Description of Computer Program	23
	8.4 Tabulation of Operating Conditions, Holdups and Dispersion Coefficients	36
	8.5 Error Analysis	42
	8.6 Nomenclature	43
	8.7 Literature References	45

1. SUMMARY

The effect of superficial gas and liquid velocities on solid, liquid, and gas volume fractions and the degree of axial mixing in a three-phase fluidized bed were investigated. Axial mixing was characterized with an axial dispersion coefficient. Air and water were introduced into a plexiglas column, 7.62-cm-ID and 152-cm high, to a bed of 0.46-cm-diam glass beads. Bed bead loadings of 2 and 3 kg, superficial air velocities varying from 1.4 to 9.3 cm/sec, and water velocities of 4.4, 6.5, 9.1, and 13.2 cm/sec were utilized in the investigation.

At constant gas superficial velocity, the liquid holdup increased and gas holdup remained approximately constant with increasing liquid velocity. At constant liquid superficial velocity, both the liquid and solids holdup decreased with increasing gas velocity.

Dispersion was measured by injecting a potassium chloride tracer at the base of the column and analyzing concentration response curves from conductivity probes installed at two positions along the column. Peclet numbers were calculated from these data by an analysis of moments and by a transfer function technique. Dispersion coefficients increased from negative values to 200 cm^2/sec with increasing liquid and gas superficial velocity, but were too irregular to be correlated with the superficial velocities or the mass of beads in the bed. Dispersion coefficients calculated with the transfer function did not agree with those calculated by the moments method and were dependent on the magnitude and number of transform variables utilized in evaluating the function.

2. INTRODUCTION

2.1 Background

A three-phase fluidized bed consists of solid particles suspended by a cocurrent flow of gas and liquid. Fluidization occurs when the drag force exerted by the fluids on the particles equals or exceeds the buoyant weight of the solids. Three-phase fluidized beds are utilized in coal liquefaction processes and bioreactors.

Several MIT Practice School groups have studied various aspects of three-phase fluidized beds. Saad *et al.* (14) derived plug flow, well-mixed, and dispersed flow models for determining the bed mass transfer coefficient and calculated mass transfer coefficients assuming the plug flow model. Burck *et al.* (5) correlated solid holdup to minimum fluidization velocity and also calculated mass transfer coefficients by assuming a dispersion coefficient in a trial and error solution. Khosrowshahi *et* *al.* (6) correlated the solid phase holdup with the Reynolds and Archimedes number and studied the hydrodynamic variables affecting minimum fluidization.

2.2 Objectives and Method of Attack

The objectives were to experimentally determine the dispersion coefficient for the column during three-phase operation as a function of liquid and gas flow rates and total mass of solids in the bed. The dispersion coefficients were calculated with two methods, the analysis of moments (7) and a transfer function method by Ostergaard and Michelsen (11). Data were obtained by injecting an electrolyte tracer at the bottom of the fluidized bed and measuring its concentration at two positions as it flowed up the column.

2.3 Theory

Plug flow dispersion models assume a fluid flow closely approximating plug flow but with some axial mixing caused by diffusion and velocity variations in the fluid. The combined effect of the two mixing activities is termed dispersion. Two techniques were employed to quantify the dispersion. The analysis of moments technique considers a portion of the fluidized bed located between two sets of tracer measuring probes. Since it is assumed that undisturbed flow occurs at the boundaries of the system, the system is termed an open vessel (7). For an open vessel, the variance of the tracer concentration versus time curve at the second measuring probe when a perfect tracer pulse is injected at the first probe position is related to the dispersion coefficient in the following manner:

$$\frac{L\sigma^2}{u} = 2\left(\frac{D}{uL}\right) + 8\left(\frac{D}{uL}\right)^2 \qquad (1)$$

where the mean residence time, $\bar{t}$ (first moment), and variance, σ^2 (second moment), are given by

$$\bar{t} = \frac{\int_0^\infty t\ C(t)\ dt}{\int_0^\infty C(t)\ dt} \qquad (2)$$

$$\sigma^2 = \frac{\int_0^\infty (t-\bar{t})^2\, C(t)\, dt}{\int_0^\infty C(t)\, dt} \tag{3}$$

Since it is impossible to inject a perfect pulse into a system, Aris (1) corrected by Bischoff (4) developed a relationship between the Peclet number and the difference between the tracer concentration history variance at the two measuring probes for an imperfect pulse injected upstream of the first measuring point. For this system, assuming no entrance or exit effects,

$$\Delta\sigma^2 = \frac{\sigma_2^2 - \sigma_1^2}{\tau^2} = 2\,\frac{D}{uL} \tag{4}$$

where:

$$\tau = \bar{t}_2 - \bar{t}_1$$

As the tracer passes the detector, the electrolyte concentration in the trailing end of the pulse approaches the detection apparatus sensitivity limit. Since the moments method integrates the tracer concentration history from zero to infinity, the uncertainty of the tracer concentration in the trailing end can introduce significant error into the calculation. This problem can be avoided if a decay function is included in the integration so that the trailing concentrations are forced to approach zero as the concentration detection limit is reached. Ostergaard and Michelsen have developed such a method for calculating the residence time and Peclet number (11). They introduce a transfer function defined as

$$F(s) = \frac{\int_0^\infty C_{z=1}(t)e^{-st}dt \Big/ \int_0^\infty C_{z=1}(t)dt}{\int_0^\infty C_{z=0}(t)e^{-st}dt \Big/ \int_0^\infty C_{z=0}(t)dt} \tag{6}$$

The mean residence time, τ', and the Peclet number may be evaluated from:

$$[\ln \frac{1}{F(s)}]^{-1} = \tau' s [\ln \frac{1}{F(s)}]^{-2} - \frac{1}{N_{Pe}} \quad (7)$$

by plotting $[\ln \frac{1}{F(s)}]^{-1}$ against $s[\ln \frac{1}{F(s)}]^{-2}$ for several values of s. The resulting line has a slope of τ' and intercept $-1/N_{Pe}$. Details of the calculation are presented in Appendix 8.1

3. APPARATUS AND PROCEDURE

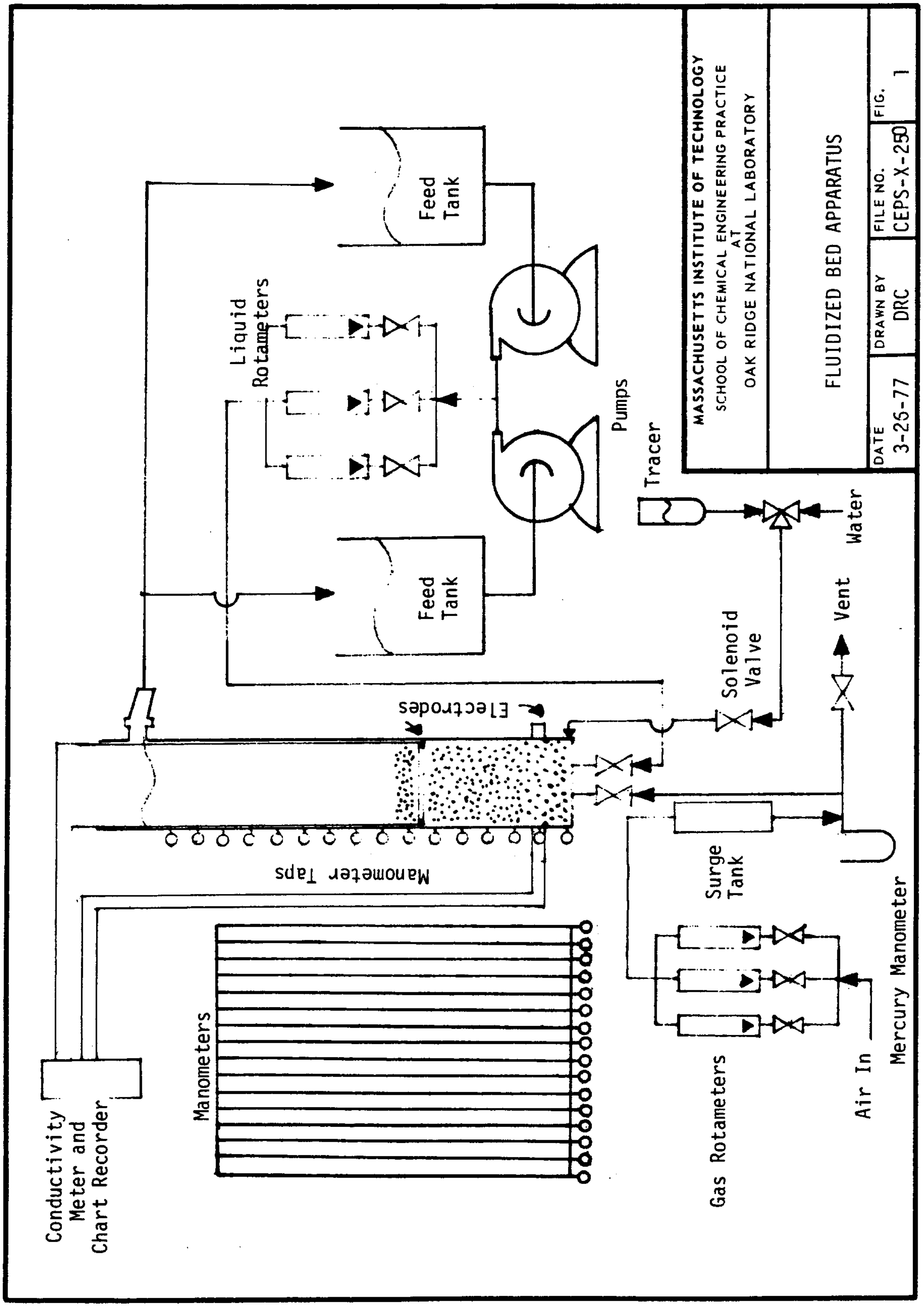
3.1 Apparatus

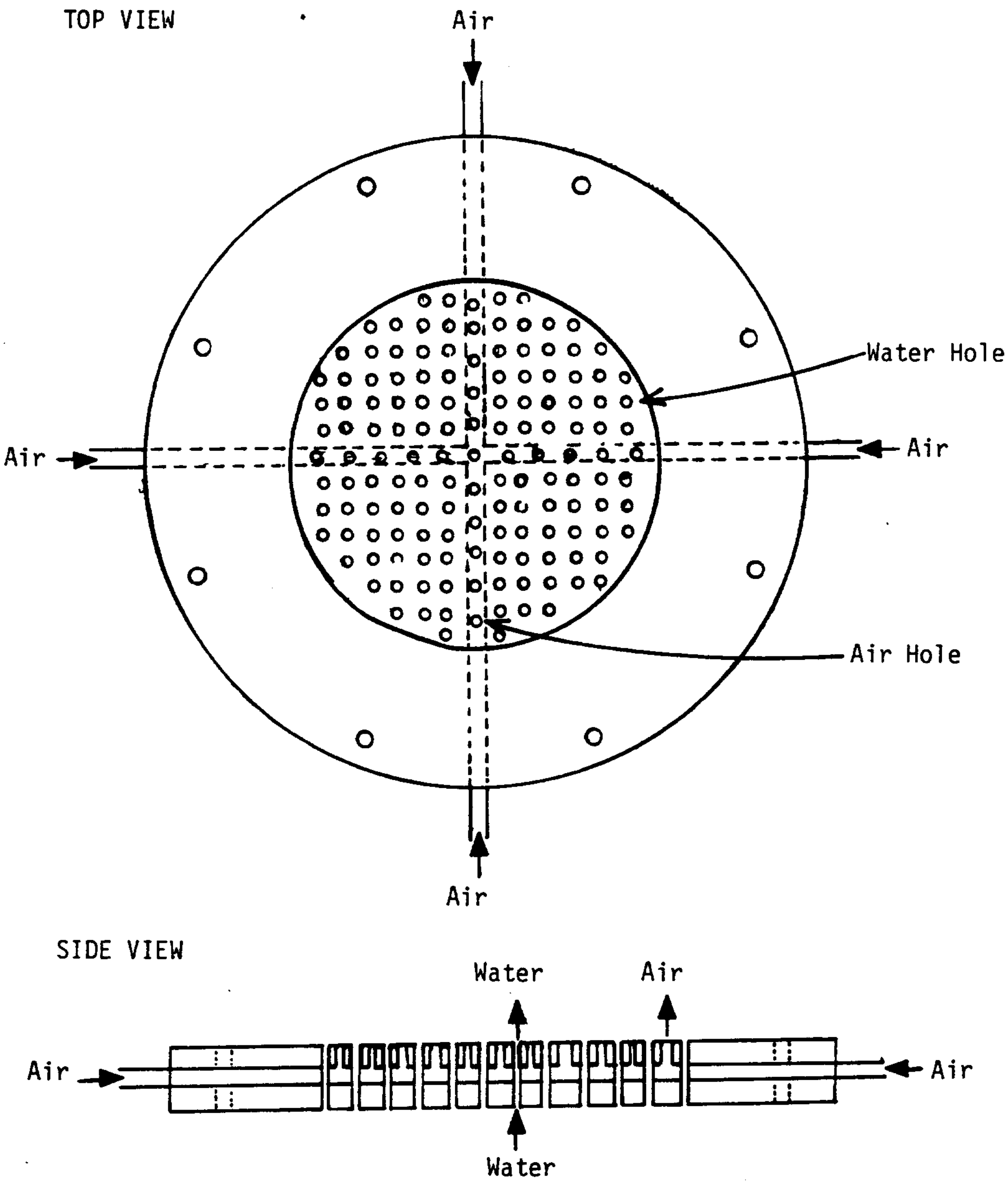
The three-phase fluidized bed consisted of a plexiglas column, 7.62-cm-ID and 152-cm high, loaded with glass beads fluidized with water and air (Fig. 1). The static bed was supported by the liquid and gas distributor, a plexiglas disc (Fig. 2). Air flowed through a gas rotameter through a surge tank and into the column through the liquid and gas distributor. The air and water flowed cocurrently upwards through the column. The water was recycled to the feed tank while the exit air was released to the atmosphere. Outlets to sixteen manometers were spaced 8-cm apart on the column.

Axial mixing was detected by injecting a tracer, potassium chloride, and measuring the conductivity between two electrodes. One pair of electrodes was located 7 cm above the tracer injection port and the second pair could be moved to any desired position in the bed. Potassium chloride was injected by using a process water line to carry the tracer into the column. A timer was used to activate a solenoid which opened the tracer injection port. Normal injection time was about 1/6 sec. During the course of the experimentation, the injection port was modified. A 1/8-in. tube placed into the injection port was drilled with six 1-mm holes to distribute the potassium chloride radially through the column. The two pairs of electrodes were connected to two conductivity meters which inputted a voltage to the chart recorder. The two readings were recorded simultaneously on the chart recorder.

3.2 Procedure

The column was loaded with a known weight of solid particles and then fluidized with air and water. Pressure was measured at each tap location along the column for each particle loading and gas and liquid flow rates once the fluidized bed height had stabilized. The temperature of the water, atmospheric pressure, and the pressure drop between the gas inlet


Feed Tank
Feed Tank
Liquid Rotameters
Pumps
Tracer
Water
Solenoid Valve
Vent
Electrodes
Manometer Taps
Surge Tank
Manometers
Conductivity Meter and Chart Recorder
Gas Rotameters
Air In
Mercury Manometer
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
SCHOOL OF CHEMICAL ENGINEERING PRACTICE
AT
OAK RIDGE NATIONAL LABORATORY
FLUIDIZED BED APPARATUS
DATE 3-25-77
DRAWN BY DRC
FILE NO. CEPS-X-250
FIG. 1



MASSACHUSETTS INSTITUTE OF TECHNOLOGY
SCHOOL OF CHEMICAL ENGINEERING PRACTICE
AT
OAK RIDGE NATIONAL LABORATORY

GAS AND LIQUID DISTRIBUTOR

DATE	DRAWN BY	FILE NO.	FIG.
3-21-77	DRC	CEPS-X-250	2

and the atmosphere were also recorded. The tracer was then injected and conductivity versus time plots were obtained for each run.

Glass beads, 0.46-cm-diam, with a density of 2.24 gm/cm^3, were loaded into the bed at either 2 or 3 kg. Superficial gas velocities ranged from 1.4 to 9.3 cm/sec with four intermediate values. Superficial liquid velocities ranged from 4.4 to 13.2 cm/sec with two intermediate values. Operating conditions for each run are tabulated in Appendix 8.4.

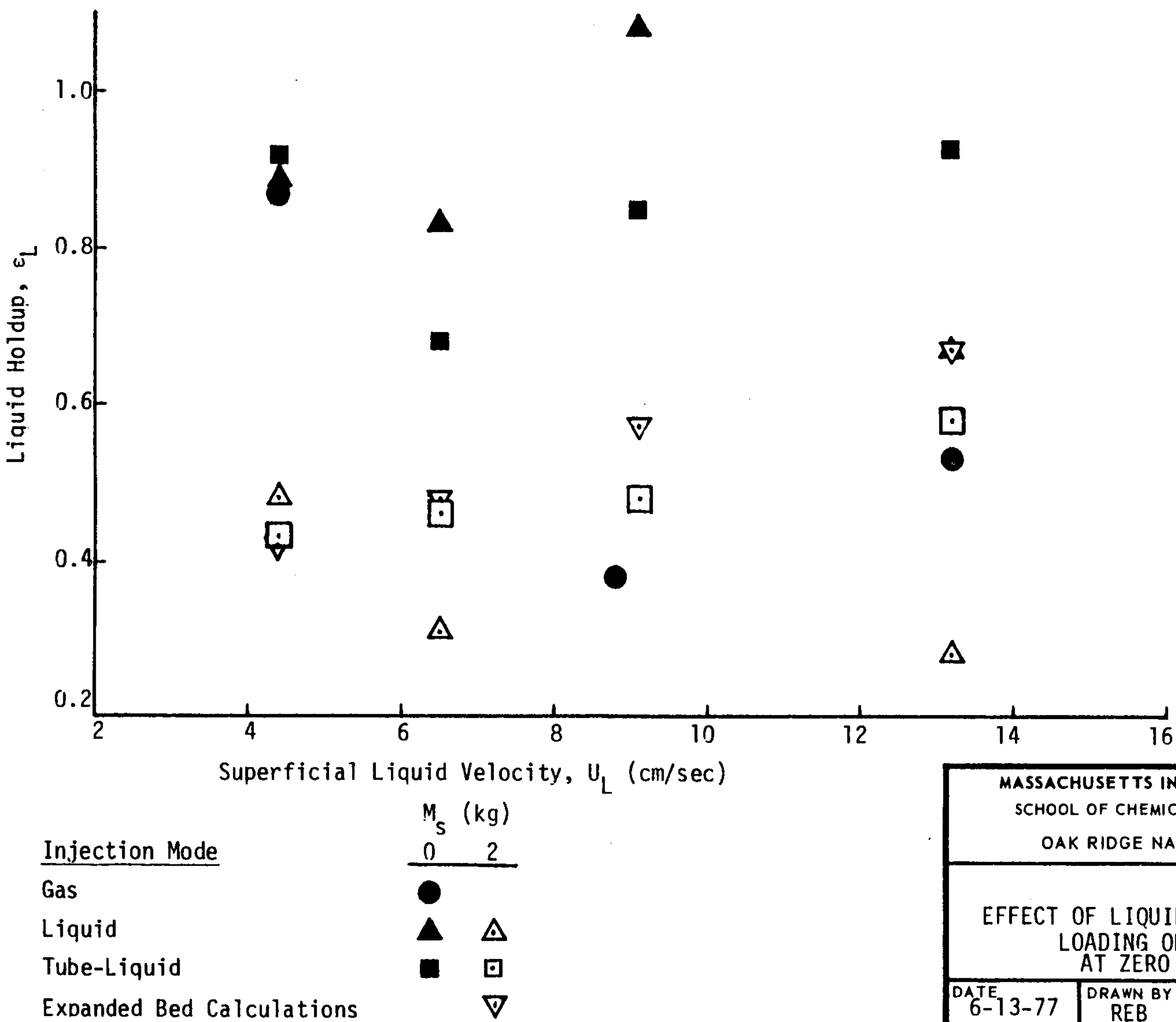
After the concentration versus time curve at each electrode was obtained, time and conductivity values from the curves were punched onto paper tape with an Elographics Digitizer.

4. RESULTS AND DISCUSSION

4.1 Liquid Volume Fraction

The liquid volume fraction (holdup) in the column was studied as a function of liquid and gas velocity and the type of tracer injection system. These studies were executed with no solids or gas, with solids but no gas, and with both solids and gas. The results are shown respectively in Figs. 3 through 6. Although these data do scatter, it is apparent that the liquid holdups from the tube injection yielded the best agreement to those holdups which were calculated from the pressure drop across the expanded bed. Thus, it would be advisable to utilize the liquid injection system which employs the tubular distributor in further studies. However, the fact that this tube extends across the bed itself is of some concern. Further studies should be executed to determine the extent to which the tubular injector affects the hydrodynamics of the bed. The tubular injection system is thought to enhance the liquid holdup agreement within the fluidized bed because of the manner and position in which the tracer enters the column. Because it enters the column at equal intervals across the column diameter, and because it is propelled outward toward the walls, the tracer is radially distributed more quickly across the column as compared to the other methods of injection. An alternative solution would be to detach the bottom set of electrodes such that it could be elevated within the bed to allow the tracer more time to become radially distributed before reaching the first measuring point.

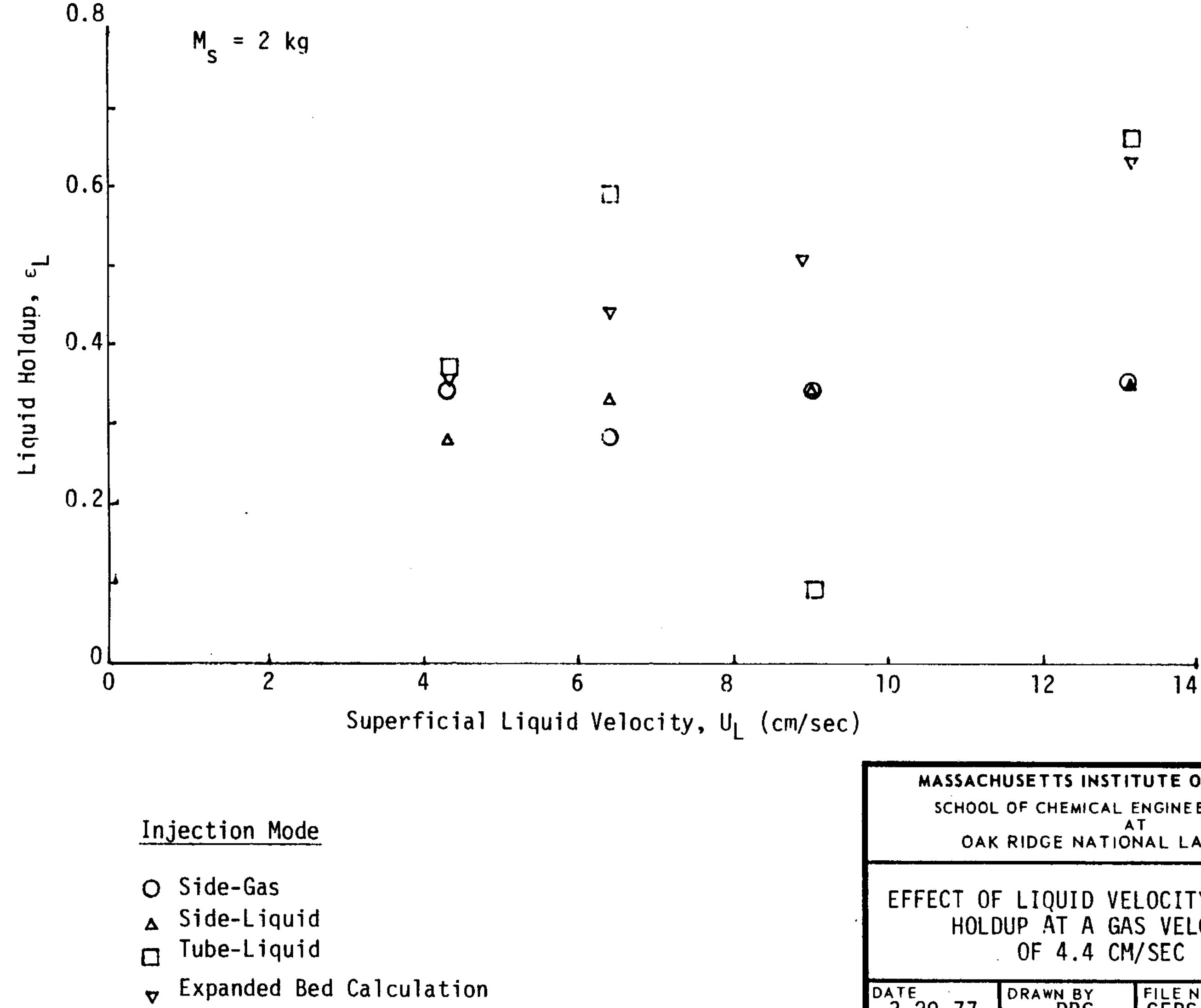
There is a slight increase in liquid holdup with liquid superficial velocity. Inspection of the solid and gas holdups (Appendix 8.4) indicates that the gas holdup is approximately constant and that the solids holdup decreases with increasing liquid velocity. Wen and Yu have shown that the drag force on fluidized particles decreases with increasing liquid holdup (16). If it is assumed that this is also true for liquid-gas fluidization, the observed increase in the liquid holdup with superficial velocity would require that the interstitial velocity increase to keep the particles fluidized. Contrary to Ostergaard and Theisen's observation (13), the solids



MASSACHUSETTS INSTITUTE OF TECHNOLOGY
SCHOOL OF CHEMICAL ENGINEERING PRACTICE
AT
OAK RIDGE NATIONAL LABORATORY

EFFECT OF LIQUID VELOCITY AND SOLIDS LOADING ON LIQUID HOLDUP AT ZERO GAS FLOW RATE

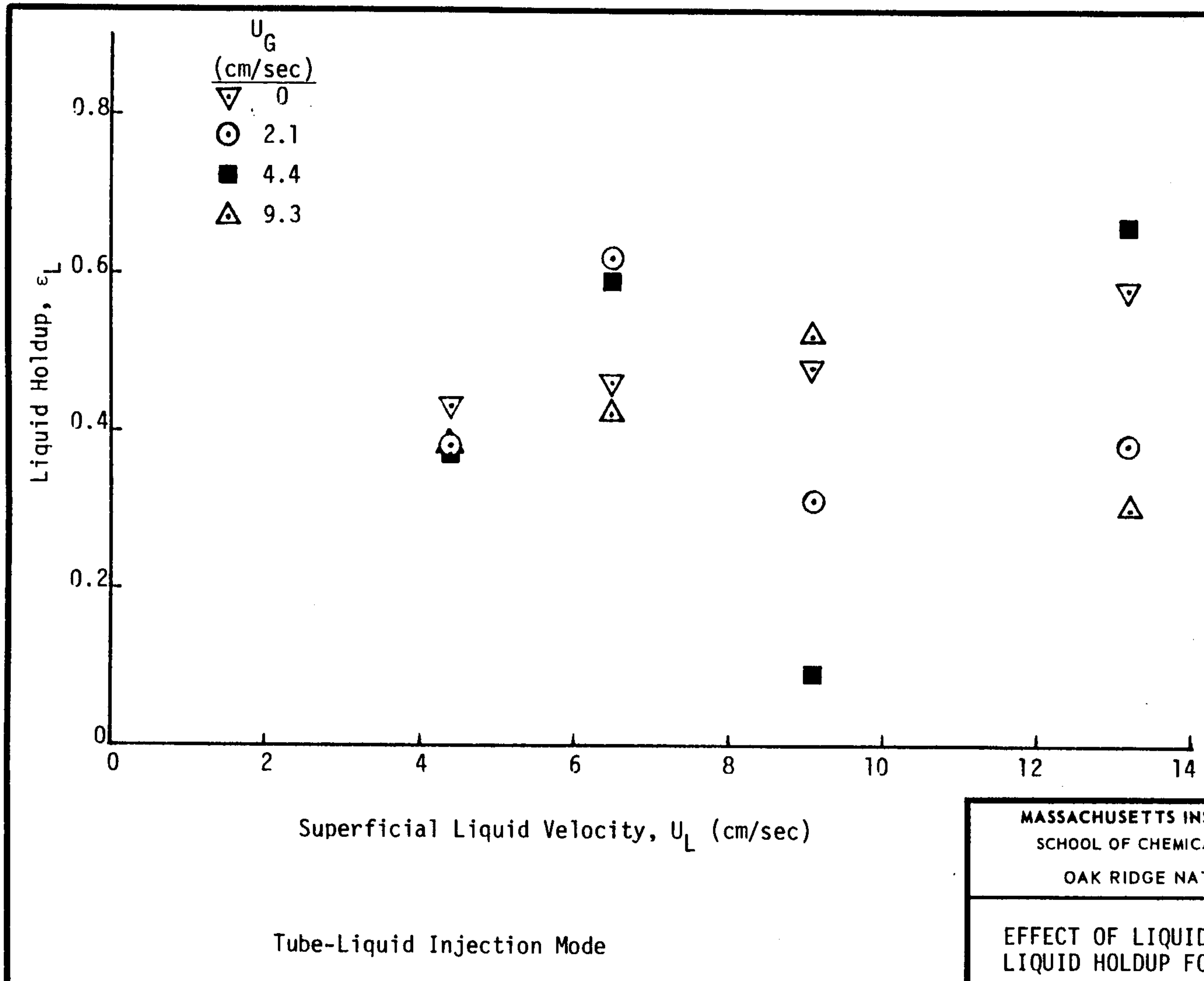
DATE	DRAWN BY	FILE NO.	FIG.
6-13-77	REB	CEPS-X-250	3



MASSACHUSETTS INSTITUTE OF TECHNOLOGY
SCHOOL OF CHEMICAL ENGINEERING PRACTICE
AT
OAK RIDGE NATIONAL LABORATORY

EFFECT OF LIQUID VELOCITY ON LIQUID HOLDUP AT A GAS VELOCITY OF 4.4 CM/SEC

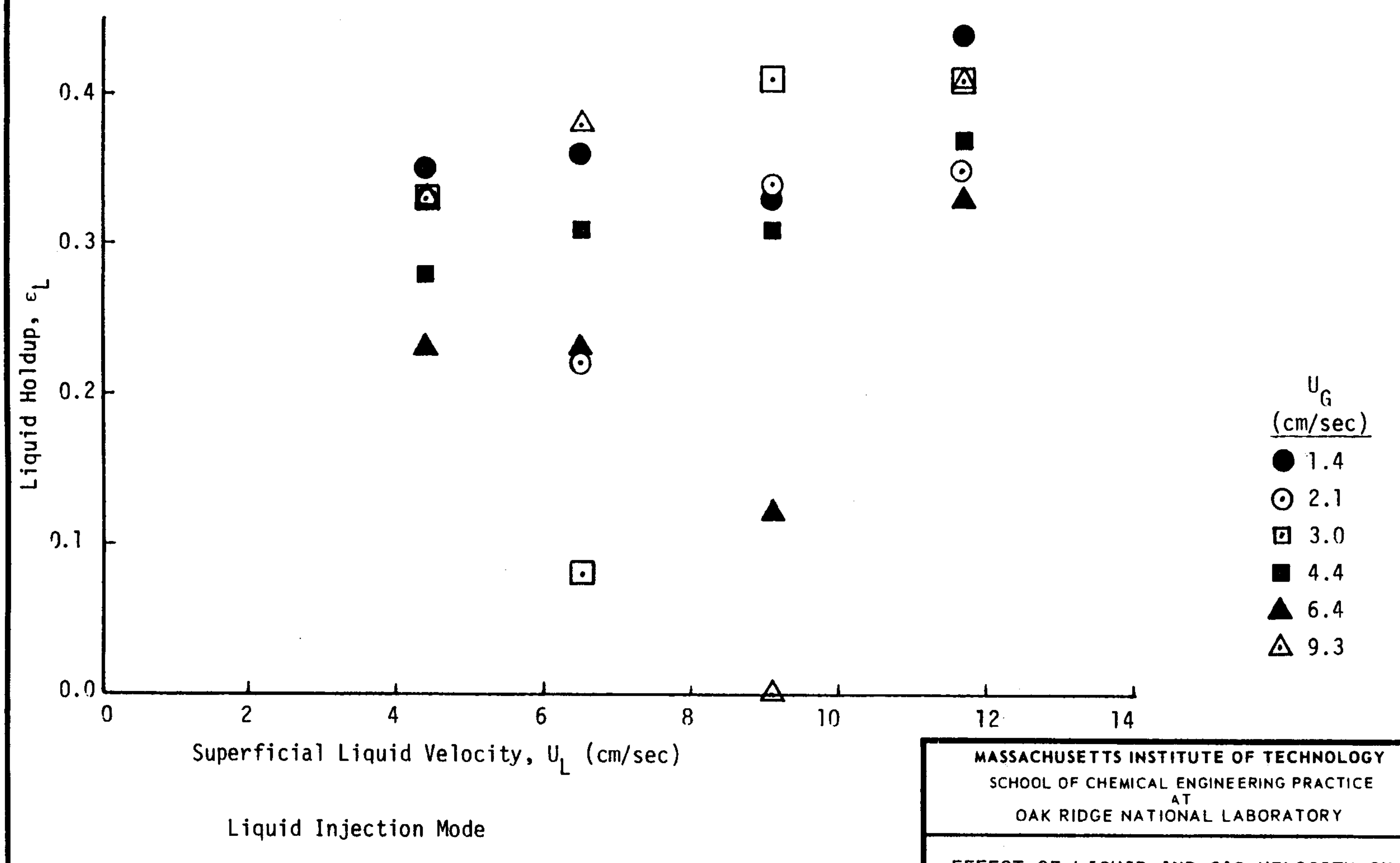
DATE	DRAWN BY	FILE NO.	FIG.
3-20-77	DRC	CEPS-X-25[illegible]	4



MASSACHUSETTS INSTITUTE OF TECHNOLOGY
SCHOOL OF CHEMICAL ENGINEERING PRACTICE
AT
OAK RIDGE NATIONAL LABORATORY

EFFECT OF LIQUID AND GAS VELOCITY ON LIQUID HOLDUP FOR 2 KG SOLIDS LOADING

DATE	DRAWN BY	FILE NO.	FIG.
6-13-77	REB	CEPS-X-250	5



MASSACHUSETTS INSTITUTE OF TECHNOLOGY			
SCHOOL OF CHEMICAL ENGINEERING PRACTICE AT OAK RIDGE NATIONAL LABORATORY			
EFFECT OF LIQUID AND GAS VELOCITY ON LIQUID HOLDUP FOR 3 KG SOLIDS LOADING			
DATE 6-13-77	DRAWN BY REB	FILE NO. CEPS-X-250	FIG. 6

holdup did not increase with the introduction of gas into the bed. Thus, if a uniform distribution of solids is assumed, the distance between spheres increased with the gas and liquid superficial velocity.

4.2 Dispersion Coefficients

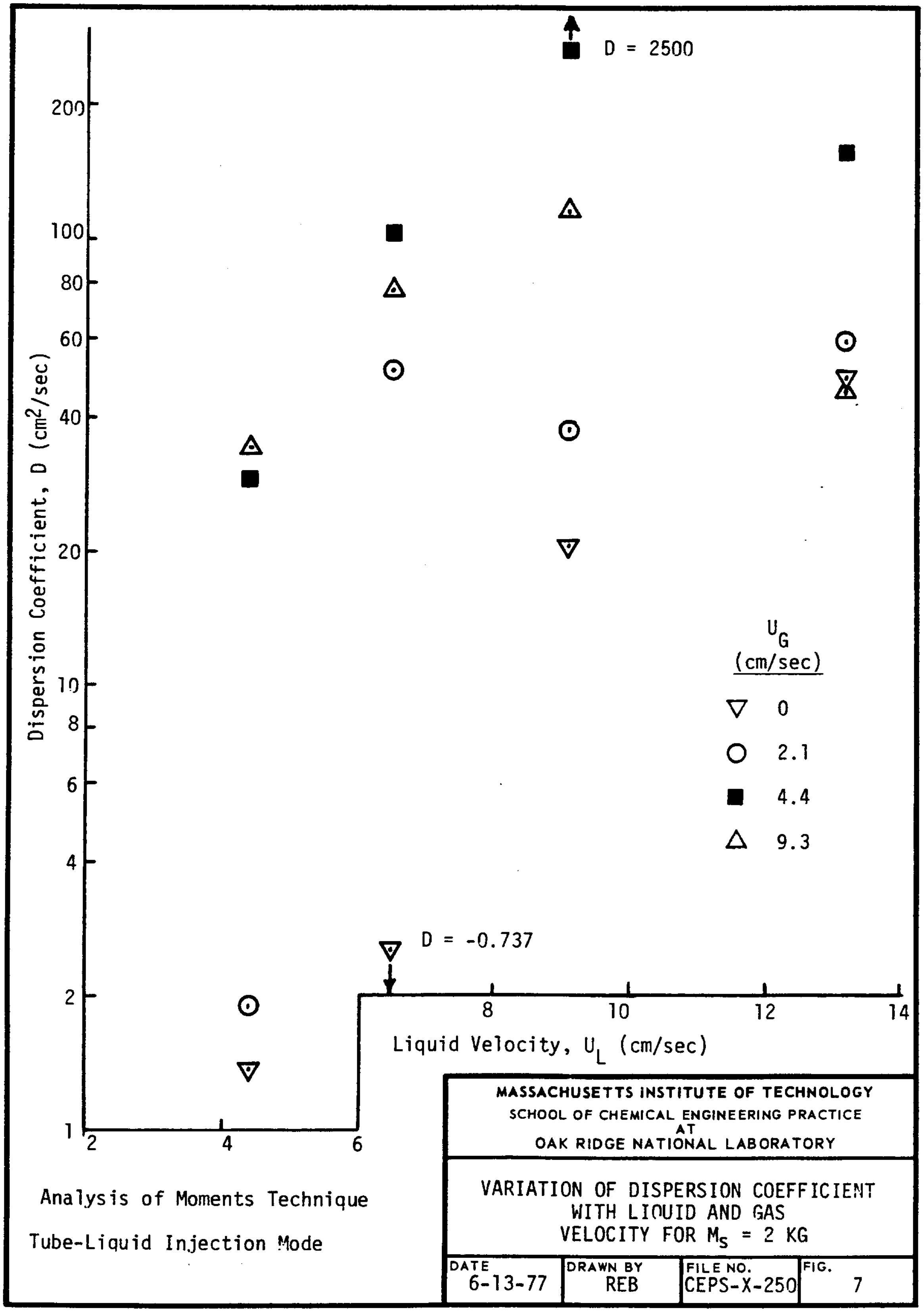
Dispersion coefficients, calculated by the method of moments, are presented in Figs. 7 and 8 for two and three kilogram loadings respectively. The tracer was injected by water from the tube for Fig. 7 data and from the wall port for the Fig. 8 data. An estimate of the uncertainty associated with the dispersion coefficients is presented in Appendix 8.5. In general, the dispersion increases with liquid superficial velocity at low gas velocities. The decrease in dispersion at high liquid velocities (e.g., U_L of 9.1 for U_G of 2.1, Fig. 7) is interesting since it might represent a transition in fluid flow through the bed. However, the data at this time are too inconsistent to substantiate such an interpretation. For example, with no particles in the bed and no gas flow rate, the dispersion coefficient drops from 45 to -12.4 between a liquid velocity of 4.4 and 6.5 cm/sec (Appendix 8.4, tube-liquid injection mode). However, the laminar to turbulent flow transition in this column would occur at 2.6 cm/sec if entrance effects are neglected (3).

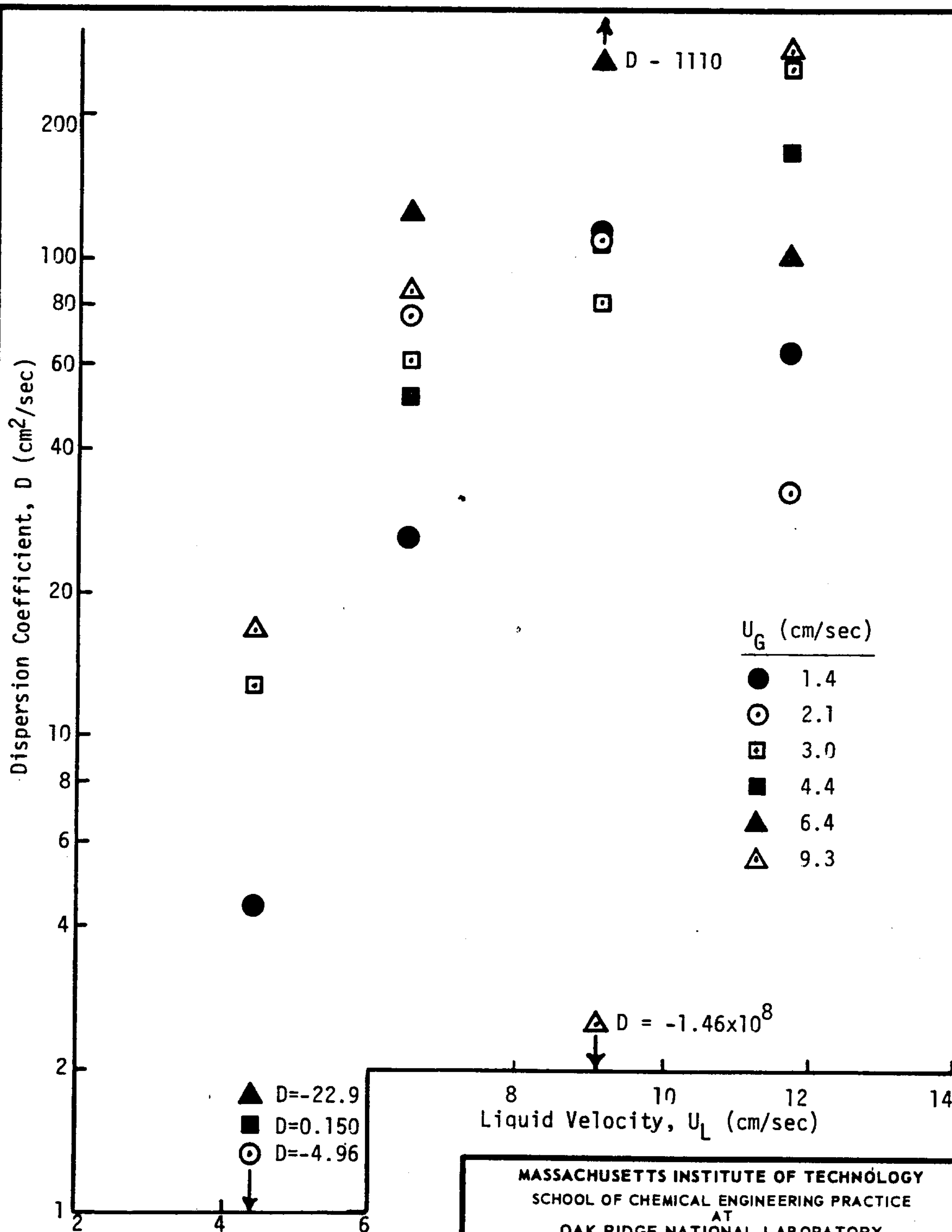
Dispersion coefficients calculated with the transform method are presented along with those calculated by the moments method in Table 1. There is vast disagreement between dispersion coefficients calculated by the two methods. The source of disagreement between the two methods was not resolved. However, it was observed that the dispersion coefficient is strongly dependent on the "s" values utilized in evaluation of Eq. (6). All transform method dispersion coefficients in Table 1 were calculated with ten "s" values ranging from 0.01 to 5.12, each a factor of two greater than the previous value. Changing the "s" values for the same tracer concentration data alters the slope and intercept of the least squares line calculated to evaluate the residence time and Peclet number from Eq. (7).

5. CONCLUSIONS

1. Of the three tracer injection methods, liquid injection through the perforated tube provides holdup values closest to those calculated from the pressure drop across the bed.

2. For a constant gas superficial velocity, the liquid holdup increases and the gas holdup remains approximately constant with increasing liquid superficial velocity.


D = 2500
Dispersion Coefficient, D (cm²/sec)
200
100
80
60
40
20
10
8
6
4
2
1
U_G
(cm/sec)
0
2.1
4.4
9.3
D = -0.737
2
4
6
8
10
12
14
Liquid Velocity, U_L (cm/sec)
Analysis of Moments Technique
Tube-Liquid Injection Mode
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
SCHOOL OF CHEMICAL ENGINEERING PRACTICE
AT
OAK RIDGE NATIONAL LABORATORY
VARIATION OF DISPERSION COEFFICIENT
WITH LIQUID AND GAS
VELOCITY FOR M_S = 2 KG
DATE 6-13-77
DRAWN BY REB
FILE NO. CEPS-X-250
FIG. 7



Analysis of Moments Technique
Liquid Injection Mode

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
SCHOOL OF CHEMICAL ENGINEERING PRACTICE
AT
OAK RIDGE NATIONAL LABORATORY

VARIATION OF DISPERSION COEFFICIENT WITH LIQUID AND GAS VELOCITY FOR M_s = 3 KG

DATE	DRAWN BY	FILE NO.	FIG.
6-13-77	REB	CEPS-X-250	8

Table 1. Comparison of Dispersion Coefficient Calculation Methods

M_s = 2 kg

Tube-Liquid Injection

Superficial Velocities (cm/sec) U_L	U_G	Dispersion Coefficient (cm^2/sec) Moments Method	Transform Method
4.4	0	1.4	10.9
	2.1	9.1	114.6
	4.4	29.2	243.8
	9.3	34.2	156.3
6.5	0	-0.7	18.4
	2.1	51.2	86.9
	4.4	103.0	247.9
	9.3	77.9	126.7
9.1	0	20.6	220.1
	2.1	37.6	46.4
	4.4	2497.1	-543.9
	9.3	117.0	184.0
13.2	0	48.9	509,604.1
	2.1	59.3	139.7
	4.4	157.6	561.9
	9.3	-45.8	-47.6

3. For a constant liquid superficial velocity, the liquid and solid holdups decrease with increasing gas superficial velocity.

4. Dispersion coefficients calculated by the method of moments are less erratic than those calculated by the transform method and increase with increasing liquid superficial velocity.

5. Dispersion coefficients calculated by the transform method depend on the choice of the Laplace transform variable.

6. RECOMMENDATIONS

1. Install a water regulator on the tracer injection line to provide a consistent injection pressure.

2. Install vertically adjustable electrodes at the bottom of the column.

3. Resolve the dependence of the transform method on the choice of the transform value by evaluating known distribution functions rather than experimental concentration data.

7. ACKNOWLEDGMENTS

We would like to thank J.S. Watson for his helpful suggestions, J.M. Begovich for his continuing guidance, and S.D. Clinton for his assistance with the computer. We are also grateful to the Metals and Ceramics Division for the use of their Digitizer.

8. APPENDIX

8.1 Derivation of Transfer Function (11)

The transfer function is derived from the mass balance for differential volume. Component i enters a differential volume by convection and diffusion and leaves by the same mechanisms. If a constant axial dispersion coefficient, D, and a constant absolute linear flow velocity, u, are assumed, the general mass balance is:

$$\frac{\partial C_i}{\partial t} = D\frac{\partial^2 C_i}{\partial x^2} - u\frac{\partial C_i}{\partial x} \tag{8}$$

Letting $z = x/L$ where L is the distance between the measuring points or detection electrodes and rearranging gives

$$\frac{L}{u}\frac{\partial C_i}{\partial t} - \frac{D}{uL}\frac{\partial^2 C_i}{\partial z^2} + \frac{\partial C_i}{\partial z} = 0 \tag{9}$$

Letting

$$\tau' = \frac{L}{u} \quad \text{and} \quad N_{Pe} = \frac{uL}{D}$$

then,

$$\tau'\frac{\partial C_i}{\partial t} - \frac{1}{N_{Pe}}\frac{\partial^2 C_i}{\partial z^2} + \frac{\partial C_i}{\partial z} = 0 \tag{10}$$

Taking the Laplace transform of Eq. (10) with respect to the time variable, noting the initial condition that $C = 0$ at $t = 0$ yields

$$\tau' s\bar{C} - \frac{1}{N_{Pe}}\frac{d^2\bar{C}}{dz^2} + \frac{d\bar{C}}{dz} = 0 \tag{11}$$

The solution to this second order differential equation is

$$\overline{C} = A \exp\left[\frac{N_{Pe} + \sqrt{N_{Pe}^2 + 4N_{Pe}\tau's}}{2} z\right] + B\exp\left[\frac{N_{Pe} - \sqrt{N_{Pe}^2 + 4N_{Pe}\tau's}}{2} z\right] \quad (12)$$

By inspection (assuming z and s positive) the exponent in the first term is always positive while that in the second is always negative. Therefore to satisfy the boundary condition that $c \rightarrow 0$ as $z \rightarrow \infty$, the constant A must be zero. Hence,

$$\overline{C} = B \exp\left[\frac{N_{Pe} - \sqrt{N_{Pe}^2 + 4N_{Pe}\tau's}}{2} z\right] \quad (13)$$

The second boundary condition is much harder to formulate and is not clear. Therefore, to alleviate this problem, a ratio of $\overline{C}$ at two different values of z is employed. Taking the ratio of $\overline{C}$ at the second electrode (z = 1) to $\overline{C}$ at the first electrode (z = 0) yields

$$F(s) = \frac{\overline{C}_{z=1}}{\overline{C}_{z=0}} = \exp\left[\frac{N_{Pe} - \sqrt{N_{Pe}^2 + 4N_{Pe}\tau's}}{2}\right] \quad (14)$$

Rearranging Eq. (14) gives

$$N_{Pe} - 2\ln F(s) = (N_{Pe}^2 + 4N_{Pe}\tau's)^{1/2} \quad (15)$$

Squaring both sides and dividing through by $N_{Pe}[\ln \frac{1}{F(s)}]^2$ gives

$$[\ln \frac{1}{F(s)}]^{-1} = \tau's[\ln \frac{1}{F(s)}]^{-2} - \frac{1}{N_{Pe}} \quad (7)$$

The transfer function, F(s), can be evaluated from the definition of a Laplace transformation, i.e.,

$$\overline{C}_i(s) = \int_0^\infty C_i(t)\, e^{-st}\, dt \quad (16)$$

Substitution of Eq. (16) into the definition of the transfer function in Eq. (14) yields

$$F(s) = \frac{\overline{C}_{z=1}}{\overline{C}_{z=0}} = \frac{\int_0^\infty C_{z=1}(t)e^{-st}dt}{\int_0^\infty C_{z=0}(t)\ e^{-st}dt} \tag{17}$$

Normalizing both the numerator and the denominator results in a form in which F(s) can be calculated from experimental data:

$$F(s) = \frac{\int_0^\infty C_{z=1}(t)e^{-st}dt \Big/ \int_0^\infty C_{z=1}(t)dt}{\int_0^\infty C_{z=0}(t)e^{-st}dt \Big/ \int_0^\infty C_{z=0}(t)dt} \tag{6}$$

From Eq. (7) it can be seen that plotting $[\ln\frac{1}{F(s)}]^{-1}$ vs $s[\ln\frac{1}{F(s)}]^{-2}$ for F(s) evaluated with several values of arbitrary s results in a straight line with slope τ' and intercept $-1/N_{Pe}$.

The degree to which the points of the above plot adhere to a straight line is an indication of the degree to which the model describes the physical system.

8.2 Calculation of Holdups

Holdups are volume fractions and must sum to one:

$$\varepsilon_G + \varepsilon_L + \varepsilon_S = 1 \tag{18}$$

The difference in pressure between the top and bottom of the bed is

$$\Delta P = (\varepsilon_G\rho_G + \varepsilon_L\rho_L + \varepsilon_S\rho_S)gH \tag{19}$$

The pressure drop through the bed was obtained with water manometer readings between the top and bottom of the bed:

$$\Delta P = \rho_L g(H + \Delta H) \tag{20}$$

where ΔH is the difference in manometer readings and H is the bed height. Combination of Eqs. (19) and (20) gives

$$\rho_L(H + \Delta H) = H(\varepsilon_G\rho_G + \varepsilon_L\rho_L + \varepsilon_S\rho_S) \tag{21}$$

The solids holdup is the ratio of mass of solids actually present to the mass of solids necessary to occupy the entire volume of the bed.

$$\varepsilon_S = \frac{M_S}{AH\rho_S} \tag{22}$$

The liquid holdup is the ratio of the superficial to interstitial liquid velocities:

$$\varepsilon_L = \frac{U_L\tau}{L} \tag{23}$$

Holdups are calculated by the computer program both by combining Eqs. (18), (21), and (22) and Eqs. (18), (21), and (23).

8.3 Description of Computer Program

A computer program was prepared for the PDP-10 (TRACER-F4) and PDP-11 (TRACER-FTN) to calculate dispersion coefficients from conductivity data.

8.3.1 Input Parameters

A disc file contains a list of runs to be analyzed. A run number is read from the list and used to compose a data file name from which information pertinent to the run is read. The data contained in the composed file are listed in Table 2.

Table 2. Data File Entries

Run number
Liquid and gas rotameters used
Readings on rotameters
Liquid temperature
Fluidized bed height
Pressure drop through the column
Barometer reading
Difference in water manometer readings through the bed for each pair of conductivity curves
Conductivity data:
Number of points
Baseline (conductivity with no tracer)
Time at each point
Conductivity value at each point
Mass of solids
Particle size and density

8.3.2 Calculation of Flow Rates, Viscosities, and Densities

Rotameter scale readings are converted to volumetric flow rates according to the calibrations appropriate to the rotameters used. If a rotameter number is indicated for which there is no calibration, an error message is printed and execution proceeds to the next run. The message indicates the run number so that the error can be corrected for rerun.

Water viscosity and density and air viscosity are determined as functions of temperature by

$$\mu_L = 0.013508 - 0.00017445\ T \qquad (24)$$

$$\rho_L = 1.00401 - 0.00028\ T \qquad (25)$$

$$\mu_G = 0.0001718 + 0.000000475\ T \qquad (26)$$

The pressure drop through the bed is calculated using manometer readings and is used to determine the pressure at the middle of the bed height for calculation of the gas density.

8.3.3 Statistical Evaluation for the Peclet Number

The method of least squares is used to determine the best straight line for evaluation of the transfer function from the plot of

$$[\ln \frac{1}{F(s)}]^{-1} = \tau' s[\ln \frac{1}{F(s)}]^{-2} - \frac{1}{N_{Pe}} \qquad (7)$$

for several values of s.

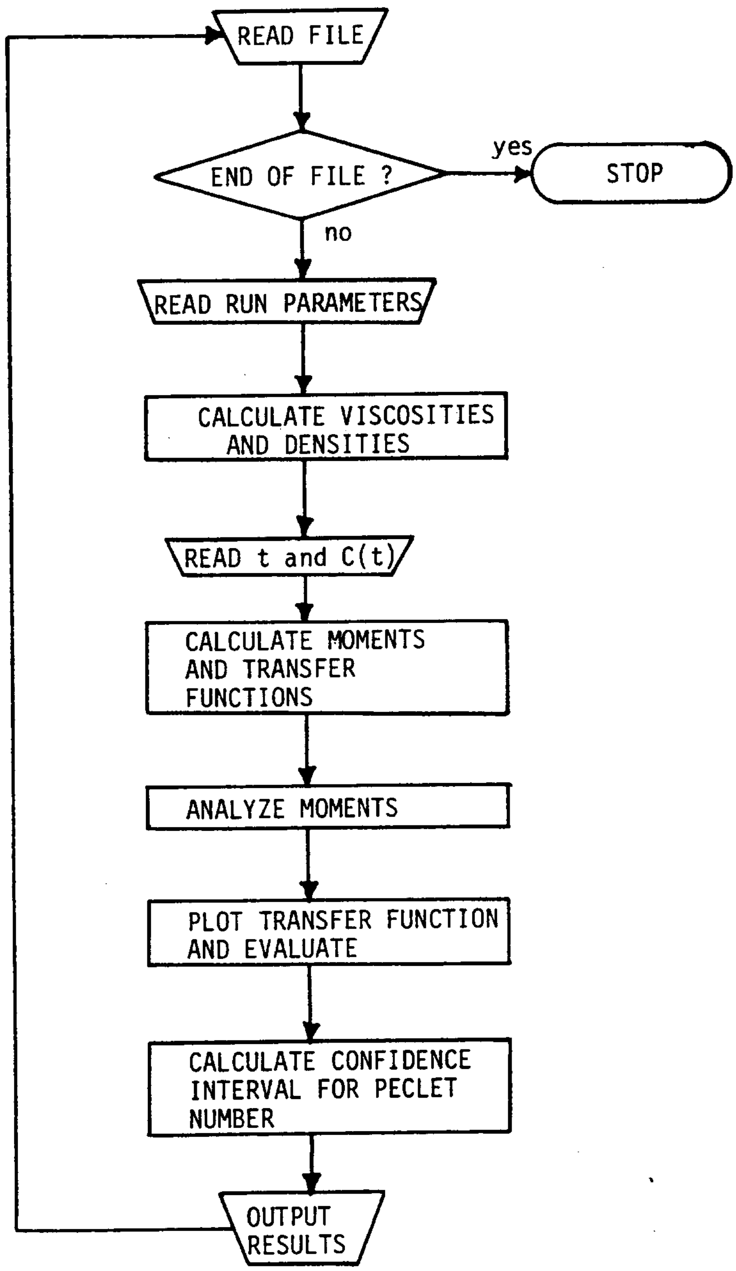
A 95% confidence interval about the y intercept, $-\frac{1}{N_{Pe}}$, is calculated by

$$\left[\frac{\frac{1}{n}\Sigma x^2}{\Sigma x^2 - \frac{1}{n}(\Sigma x)^2}\right]^{1/2} \left[\frac{\Sigma(y - \hat{y})^2}{n - 2}\right]^{1/2} t \qquad (27)$$

where $\hat{y}$ is the y value predicted by the least squares line. For n data points there are n-2 degrees of freedom since the least square line fixes two points. For a two-sided 95% confidence interval with eight degrees of freedom (ten points), the value of t, from Student's distribution, is 2.306.

8.3.4 Program Flow Chart

The program flow chart is shown in Fig. 9.



MASSACHUSETTS INSTITUTE OF TECHNOLOGY
SCHOOL OF CHEMICAL ENGINEERING PRACTICE
AT
OAK RIDGE NATIONAL LABORATORY

PROGRAM FLOW CHART

DATE	DRAWN BY	FILE NO.	FIG.
6-13-77	REB	CEPS-X-250	9

8.3.5 Program Listing

```
C      TRACER.FTN
C           FLUIDIZED BED VERSION FOR TRACER EXPERIMENT
C
C      CALCULATIONS OF THE VESSEL DISPERSION NUMBER, VDN(D/UL)
C      AND THE LIQUID PHASE AXIAL DISPERSION COEFFICIENT (D)
C
C      CALCULATIONS ARE MADE BOTH BY ANALYSIS OF MOMENTS AND
C      BY A TRANSFER FUNCTION.
C      REFERENCES: O.LEVENSPIEL, CHEMICAL REACTION ENGINEERING,
C      SECOND ED., CHAPTER 9., K. OSTERGAARD AND M.L. MICHELSON,
C      ON THE USE OF THE IMPERFECT TRACER PULSE METHOD FOR
C      "DETERMINATION OF HOLD-UP AND AXIAL MIXING," THE CANADIAN
C      JOURNAL OF CHEMICAL ENGINEERING, VOL 47, APRIL 1969.
C      ALL UNITS ARE IN CGS SYSTEM.
C
C   TABLE OF SYMBOLS (ALPHABETICAL)
C
C   B1       BASELINE ORDINATE OF FIRST PEAK
C   B2       BASELINE ORDINATE OF SECOND PEAK
C   C        AVERAGE CONCENTRATION READING FOR THE TIME INTERVAL
C   CDELT    CONCENTRATION TIMES TIME INTERVAL
C   CHS      CHART SPEED (IN/MIN)
C   CONINT   95% CONFIDENCE INTERVAL FOR Y INTERCEPT
C   CPB      CONCENTRATION PLUS BASELINE (ABSOLUTE READING)
C   CS       CONCENTRATION TIMES ARBITRARY S
C   CSA      CROSS SECTIONAL AREA OF COLUMN (SQ CM)
C   C1S      CS FOR THE FIRST PEAK
C   D        REAL LIQUID PHASE AXIAL DISPERSION COEFFICIENT (CM SQ/SEC)
C   DELH     DIFFERENCE OF MANOMETER READINGS THROUGH BED (CM H2O)
C   DELP     PRESSURE DROP THROUGH BED (MM HG)
C   DELPHG   PRESSURE DROP THROUGH ENTIRE COLUMN, READ FROM HG MANOMETER (MM HG)
C   DELSIG   SIGSQ2 - SIGSQ1
C   DELTAT   TIME INTERVAL
C   DELTBR   (MEAN RESIDENCE TIME)2 - (MEAN RESIDENCE TIME)1
C   DENOM    DENOMINATOR FOR THE LEAST SQUARES SLOPE
C   DIA      COLUMN DIAMETER (CM)
C   DIMSIG   DIMENSIONLESS VARIANCE
C   DP       PARTICLE DIAMETER (CM)
C   DPTF     TRANSFER FUNCTION DISPERSION COEFFICIENT
C   EG       GAS VOLUME FRACTION WITH RESPECT TO TOTAL VOLUME
C   EGPTF    GAS VOLUME FRACTION - TRANSFER FUNCTION, TOTAL VOLUME
C   EL       LIQUID VOLUME FRACTION WITH RESPECT TO TOTAL VOLUME
C   ELPTF    LIQUID VOLUME FRACTION - TRANSFER FUNCTION, TOTAL VOLUME
C   E1G      GAS VOLUME FRACTION NOT USING MEAN RESIDENCE TIME
C   E1L      LIQUID VOLUME FRACTION NOT USING MEAN RESIDENCE TIME
C   E1S      SOLIDS VOLUME FRACTION NOT USING MEAN RESIDENCE TIME
C   F        TRANSFER FUNCTION F(S)
C   GASPER   GAS FLOW RATE (ROTAMETER SCALE READING)
```

```
C  H        BED HEIGHT (CM)
C  I        DO LOOP COUNTER
C  IRDGRT   I READ GAS RATE - GAS ROTAMETER NUMBER
C  IRDLRT   I READ LIQUID RATE - LIQUID ROTAMETER NUMBER
C  ITT      PEAK COUNTER (WORKING ON FIRST OR SECOND PEAK)
C  L        DISTANCE BETWEEN ELECTRODES (CM)
C  L1       CHARACTERS FOR NAMING OF DATA INPUT FILE
C  L2       CHARACTERS OF INPUT FILE NUMBER
C  M        TOTAL NUMBER OF CHARACTERS FOR DATA FILE NAME
C  MUG      GAS VISCOSITY (POISE)
C  MUL      LIQUID VISCOSITY (POISE)
C  NCONT    RESPONSE FOR DESIRE WHETHER TO CONTINUE
C  NO1      NUMBER OF POINTS IN FIRST PEAK
C  NO2      NUMBER OF POINTS IN SECOND PEAK
C  NRUN     RUN NUMBER
C  NU       NUMBER OF DIGITS IN NUMBER OF INPUT DATA FILE
C  NUM      NUMERATOR FOR LEAST SQUARES SLOPE
C  PATM     ATMOSPHERIC PRESSURE AT THE TIME OF THE RUN (MM HG)
C  PE       PECLET NUMBER (VL*L/D)
C  PEDP     PECLET NUMBER BASED ON PARTICLE DIAMETER
C  PEPTF    PECLET NUMBER FROM TRANSFER FUNCTION
C  PEPTFD   TRANSFER FUNCTION PECLET NUMBER BASED ON PARTICLE DIAMETER
C  PEPTFH   HIGH VALUE OF 95% CONFIDENCE INTERVAL
C  PEPTFL   LOW VALUE OF 95% CONFIDENCE INTERVAL
C  R        CORRELATION COEFFICIENT (LEAST SQUARES FIT)
C  REG      GAS REYNOLDS NUMBER BASED ON PARTICLE DIAMETER
C  REGPTF   GAS REYNOLDS NUMBER BASED ON PARTICLE DIAMETER (TRANSFER FUNCTION)
C  REGS     GAS REYNOLDS NUMBER USING SUPERFICIAL VELOCITY
C  REL      LIQUID REYNOLDS NUMBER BASED ON PARTICLE DIAMETER
C  RELPTF   LIQUID REYNOLDS NUMBER BASED ON PARTICLE DIAMETER (TRANSFER FUNCTION)
C  RELS     LIQUID REYNOLDS NUMBER USING SUPERFICIAL VELOCITY
C  RHOG     GAS DENSITY (GM/CC)
C  RHOL     LIQUID DENSITY (GM/CC)
C  RHOS     SOLID DENSITY (GM/CC)
C  RSQ      CORRELATION NUMBER IN LEAST SQUARES FIT (SQUARE OF R)
C  S        ARBITRARY NUMBERS FOR TRANSFER FUNCTION
C  SCES#T   ACCUMULATOR FOR INTEGRATING LAPLACE TRANSFORM OF CONCENTRATION
C  SD       STANDARD DEVIATION (ACTUAL AND PREDICTED VALUES ON
C             TRANSFER FUNCTION PLOT
C  SIGSQ    SIGMA SQUARED, VARIANCE
C  SIGSQ1   VARIANCE OF FIRST PEAK
C  SMASS    SOLIDS LOADING (GRAMS)
C  SUMC     INTEGRAL OF CONCENTRATION
C  SUMDIF   SUM OF SQUARED DIFFERENCES FOR SD
C  SUMTC    INTEGRAL OF C*TIME          (FIRST MOMENT)
C  SUMTTC   INTEGRAL OF C*TIME SQUARED  (SECOND MOMENT)
C  SUMX     SUM OF X TO FIND XMEAN IN LEAST SQUARES FIT
C  SUMXX    SUM OF X SQUARED TO FIND VARIANCE OF X
C  SUMXY    SUM OF X*Y FOR COVARIANCE
C  SUMY     SUM OF Y TO FIND YMEAN
C  SUMYY    SUM OF Y SQUARED TO FIND VARIANCE OF Y
C  T        ABSCISSA OF PEAK IN CHART SQUARES (10 SQUARES/IN)
```

```
      C   TAU      SLOPE OF TRANSFER FUNCTION PLOT, MEAN RESIDENCE TIME
      C   TAVG     MIDPOINT OF TIME INTERVAL
      C   TBAR     MEAN RESIDENCE TIME , SECOND PEAK
      C   TBAR1    FIRST PEAK
      C   TEMP     LIQUID TEMPERATURE (DEG C)
      C   TM       AVERAGE TIME IN CHART SQUARES
      C   TMEAN    AVERAGE TIME IN SECONDS
      C   V        LIQUID FLOW RATE (ROTAMETER SCALE READING)
      C   VDN      LIQUID PHASE AXIAL DISPERSION NUMBER
      C   VDNPTF   DISPERSION NUMBER FROM TRANSFER FUNCTION
      C   VG       REAL GAS VELOCITY (CM/SEC)
      C   VGPTF    GAS INTERSTITIAL VELOCITY BASED ON TRANSFER FUNCTION CALCULATION (CM/SEC)
      C   VGSUP    SUPERFICIAL GAS VELOCITY (CM/SEC)
      C   VGSUPV   SUPERFICIAL GAS FLOW RATE (CC/SEC)
      C   VL       REAL LIQUID VELOCITY (CM/SEC)
      C   VLSUP    SUPERFICIAL LIQUID VELOCITY (CM/SEC)
      C   VLSUPV   SUPERFICIAL LIQUID FLOW RATE (CC/SEC)
      C   VLPTF    LIQUID INTERSTITIAL VELOCITY BASED ON TRANSFER FUNCTION CALCULATION (CM/SEC)
      C   X        ABSCISSA OF TRANSFER FUNCTION POINTS
      C   Y        ORDINATE OF TRANSFER FUNCTION POINTS
      C   YINT     Y INTERCEPT OF TRANSFER FUNCTION PLOT
      C   YINTH    HIGH VALUE OF 95% CONFIDENCE INTERVAL
      C   YINTL    LOW VALUE OF 95% CONFIDENCE INTERVAL
      C   YPRED    VALUE PREDICTED FOR Y BY LEAST SQUARES SLOPE AND INTERCEPT
      C
0001          DIMENSION T(75),CPB(75),S(10),CS(10),C1S(10),F(10),
             *    X(10),Y(10),CDELT(75),TAVG(75)
0002          LOGICAL*1 L1(8),L2(3),L3(6)
0003          EQUIVALENCE (L2(1),L1(6))
0004          REAL L,MUG,MUL,NUM
0005          DATA DIA/7.62/,L3/'D','O','.','F','T','N'/,L1/'D',
             *    'A','T','A','.','O','O','O'/,
             *    S/0.01,0.02,0.04,0.08,0.16,0.32,0.64,1.28,2.56,5.12/
      C
0006          CSA=DIA*DIA*.785398
0007          M = 6
0008          CALL ASSIGN (2,L3,M)
0009          CALL FDBSET (2,'OLD')
0010     10   ITT=1
      C
      C   ASK FOR RUN NUMBER.  USE THIS TO MAKE A DATAFILE NAME FROM
      C   WHICH TO READ DATA.
      C
0011          READ (2,95,END=99) NU, L2
0012          M=5+NU
0013          CALL ASSIGN(1,L1,M)
0014          CALL FDBSET(1,'OLD')
      C
      C   READ PARAMETER VALUES FOR RUN.
      C
```

```
      READ(1,30)NRUN,NO1,NO2,B1,B2,V,GASPER,DELPHG,PATM,CHS
      READ (1,31) DP, RHOS, TEMP, L, H, DELH, SMASS
      READ (1,32) IRDGRT, IRDLRT
C
C  FIND THE GAS FLOW RATE FOR THE ROTAMETER USED.
C
      GO TO (11,12,11,11,11,13,11), IRDGRT
 11   TYPE 14, IRDGRT
      CALL CLOSE(1)
      GO TO 99
 12   VGSUPV = GASPER*0.53333*SQRT(749.8/(PATM+DELPHG))
      GO TO 16
 13   VGSUPV = GASPER*8.5526*SQRT(749.8/(PATM+DELPHG))
 16   VGSUP = VGSUPV/CSA
C
C  FIND THE LIQUID FLOW RATE FOR THE ROTAMETER USED.
C
      GO TO (18,19,20,17,21,17), IRDLRT
 17   TYPE 22, IRDLRT
      CALL CLOSE(1)
      GO TO 99
 18   VLSUPV = V*5.44
      GO TO 23
 19   VLSUPV = V*0.383
      GO TO 23
 20   VLSUPV = V*13.383
      GO TO 23
 21   VLSUPV = V*2.79
 23   VLSUP = VLSUPV/CSA
C
C  CALCULATE VISCOSITIES AND DENSITIES.
C
      MUL = (1.3508 - 0.017445*TEMP)*0.01
      MUG = (0.01718 + 0.0000475*TEMP)*0.01
      RHOL = 1.00401 - 0.00028*TEMP
      DELP = (DELH+H)*RHOL/1.356
      RHOG = 0.000463 *(PATM+DELPHG-0.5*DELP)/(273.15+TEMP)
C
C  AVERAGE TIME.
C
      TMEAN=L/VLSUP
      TM=TMEAN*CHS/6.0
C
C  INITIALIZE TIMES AND CONCENTRATIONS.
C
 25   DO 40 I=1,75
      T(I)=0.0
 40   CPB(I)=0.0
      IF(ITT.EQ.2)NO1=NO2
      NO1=NO1+1
```

```
      C
      C  READ TIMES AND CONCENTRATIONS.
      C
            READ (1,35)(T(I),CPB(I),I=2,NO1)


      C
      C  INITIALIZE SUMS.
      C
            SUMC=0.0
            SUMTC=0.0
            SUMTTC=0.0
            SCES1T=0.0
            SCES2T=0.0
            SCES3T=0.0
            SCES4T=0.0
            SCES5T=0.0
            SCES6T=0.0
            SCES7T=0.0
            SCES8T = 0.0
            SCES9T = 0.0
            SCES0T = 0.0

            IF(ITT.EQ.2)B1=B2
            DO 50 I = 2, NO1
            DELTAT = T(I) - T(I-1)
      C
      C  CONCENTRATION FOR A TIME INTERVAL IS THE AVERAGE OF THE VALUES
      C  AT EACH END OF THE TIME INTERVAL, MINUS THE BASELINE VALUE.
      C
            CPB(I) = CPB(I) - B1
            C = (CPB(I)+CPB(I-1))/2.
            TAVG(I) = (T(I)+T(I-1))*0.5
            CDELT(I) = C*DELTAT
            SUMC=SUMC+CDELT(I)
      C
      C  FIRST MOMENT TO FIND THE MEAN RESIDENCE TIME
      C
            SUMTC=SUMTC+CDELT(I)*TAVG(I)
      C
      C  LAPLACE TRANSFORM OF CONCENTRATION
      C
            SCES1T=SCES1T+CDELT(I)*EXP(-S(1)*TAVG(I))
            SCES2T=SCES2T+CDELT(I)*EXP(-S(2)*TAVG(I))
            SCES3T=SCES3T+CDELT(I)*EXP(-S(3)*TAVG(I))
            SCES4T=SCES4T+CDELT(I)*EXP(-S(4)*TAVG(I))
            SCES5T=SCES5T+CDELT(I)*EXP(-S(5)*TAVG(I))
            SCES6T=SCES6T+CDELT(I)*EXP(-S(6)*TAVG(I))
            SCES7T=SCES7T+CDELT(I)*EXP(-S(7)*TAVG(I))
            SCES8T = SCES8T + CDELT(I)*EXP(-S(8)*TAVG(I))
            SCES9T = SCES9T + CDELT(I)*EXP(-S(9)*TAVG(I))
       50   SCES0T = SCES0T + CDELT(I)*EXP(-S(10)*TAVG(I))
            PRINT 98, SUMC, SUMTC, SCES1T, SCES0T
            TBAR = SUMTC/SUMC
```

```
C
C   SECOND MOMENT TO FIND VARIANCE
C
        DO 52 I = 2, NO1
        SUMTTC = SUMTTC + CDELT(I)*(TAVG(I)-TBAR)*(TAVG(I)-TBAR)
 52     CONTINUE
        SIGSQ = SUMTTC/SUMC
C
C   NORMALIZE VALUES FOR LAPLACE TRANSFORM OF CONCENTRATION
C
        CS(1)=SCES1T/SUMC
        CS(2)=SCES2T/SUMC
        CS(3)=SCES3T/SUMC
        CS(4)=SCES4T/SUMC
        CS(5)=SCES5T/SUMC
        CS(6)=SCES6T/SUMC
        CS(7)=SCES7T/SUMC
        CS(8) = SCES8T/SUMC
        CS(9) = SCES9T/SUMC
        CS(10) = SCES0T/SUMC
        IF(ITT.EQ.2)GO TO 60
        PRINT 98, TBAR, SIGSQ, SUMTTC, SUMC
C
C   STORE VALUES FOR FIRST PEAK AND GO BACK TO DO SECOND PEAK.
C
        TBAR1=TBAR
        SIGSQ1=SIGSQ
        DO 51 I=1,10
 51     C1S(I)=CS(I)
        ITT=2
        GO TO 25
C
C   TAKING THE DIFFERENCE OF RESPONSES AT TWO MEASURING POINTS
C   ELIMINATES THE EFFECT OF AN IMPERFECT TRACER PULSE.
C
 60     DELSIG=SIGSQ-SIGSQ1
        DELTBR=TBAR-TBAR1
        PRINT 98, DELTBR, DELSIG, SIGSQ, TBAR
   98 FORMAT (4F12.5)
        E1S = SMASS/H/CSA/RHOS
        E1G = (DELH*RHOL/H - E1S*(RHOS-RHOL))/(RHOG - RHOL)
        E1L = 1.0 - E1S - E1G
        EL=DELTBR/TM
        EG = ((DELH+H)*RHOL/H - EL*(RHOL-RHOS) - RHOS)/(RHOG - RHOS)
        DIMSIG=DELSIG/(DELTBR*DELTBR)
C
C   DISPERSION NUMBER FROM THE SECOND MOMENT
C
        VDN= DIMSIG/2.
        PE=1.0/VDN
        VL=VLSUP/EL
        VG=VGSUP/EG
C
C   REYNOLDS NUMBERS WITH WHICH TO CORRELATE ABOVE DISPERSION NUMBER
```

```
C
      REG=RHOG*DP*VG/MUG
      REGS = RHOG*DP*VGSUP/MUG
      REL=RHOL*DP*VL/MUL
      RELS = RHOL*DP*VLSUP/MUL
      PEDP=PE*DP/L
C
C  LIQUID AXIAL DISPERSION COEFFICIENT
C
      D=VL*L*VDN
      DO 61 I=1,10
C
C  CALCULATE VALUES OF THE TRANSFER FUNCTION AND PLOT.
C
      F(I)=CS(I)/C1S(I)
      Y(I)=-1.0/ALOG(F(I))
 61   X(I)=S(I)/(ALOG(F(I))*ALOG(F(I)))
C
C  FIT A STRAIGHT LINE TO THE POINTS BY LEAST SQUARES.
C
      SUMX=0.0
      SUMY=0.0
      SUMXX=0.0
      SUMXY=0.0
      SUMYY=0.0
      DO 62 I = 1, 10
      SUMX=SUMX+X(I)
      SUMY=SUMY+Y(I)
      SUMXX=SUMXX+X(I)*X(I)
      SUMXY=SUMXY+X(I)*Y(I)
 62   SUMYY=SUMYY+Y(I)*Y(I)
      NUM=SUMXY-(SUMX*SUMY/10.0)
      DENOM=SUMXX-(SUMX*SUMX/10.0)
C
C  THE SLOPE IS THE MEAN RESIDENCE TIME.
C
      TAU=NUM/DENOM
      YINT=(SUMY-TAU*SUMX)/10.0
C
C  FIND A 95% CONFIDENCE INTERVAL ABOUT THE Y INTERCEPT.
C
      SUMDIF = 0.
      DO 63 I = 1, 10
      YPRED = YINT + TAU*X(I)
 63   SUMDIF = SUMDIF + (YPRED-Y(I))*(YPRED-Y(I))
      SD = SQRT(SUMDIF/8.)
      CONINT =  SQRT(SUMXX/10.0/DENOM)*SD*2.306
      YINTH = YINT - CONINT
      YINTL = YINT + CONINT
C
C  FIND THE CORRELATION NUMBER AND COEFFICIENT FOR THE LEAST SQUARES FIT.
C
```

```
      RSQ=NUM*NUM/((SUMXX-(SUMX*SUMX)/10.)*(SUMYY-(SUMY*SUMY)/10.))
      R=SQRT(RSQ)
C
C  FIND FLUID VELOCITIES AND HOLDUPS BASED ON THE MEAN RESIDENCE
C  TIME DETERMINED BY THE TRANSFER FUNCTION.
C
      ELPTF=TAU/TM
      EGPTF = ((DELH+H)*RHOL/H - ELPTF*(RHOL-RHOS) - RHOS)/(RHOG - RHOS)
      VLPTF=VLSUP/ELPTF
      VGPTF=VGSUP/EGPTF
C
C  THE DISPERSION NUMBER IS THE NEGATIVE OF THE Y INTERCEPT.
C
      VDNPTF=-YINT
C
C  PECLET AND REYNOLDS NUMBERS FOR THE TRANSFER FUNCTION
C
      PEPTF=-1.0/YINT
C
C  TRANSLATE THE Y INTERCEPT INTERVAL TO A 95% CONFIDENCE INTERVAL
C  ABOUT THE PECLET NUMBER FOUND BY THE TRANSFER FUNCTION.
C
      PEPTFH = -1.0/YINTL
      PEPTFL = -1.0/YINTH
      PEPTFD=PEPTF*DP/L
      DPTF=VLPTF*L*VDNPTF
      RELPTF=RHOL*DP*VLPTF/MUL
      REGPTF=RHOG*DP*VGPTF/MUG
C
C  TYPE OUT THE RESULTS.
C
      TYPE 70,NRUN,VGSUPV,VGSUP,REGS,VLSUPV,VLSUP,RELS,TMEAN,TM,E1G,E1L
      TYPE 71,VG,VL,REG,REL,TBAR1,TBAR,DELTBR,EG,EL,
     *    VDN,PE,PEDP,D
      TYPE 72,VGPTF,VLPTF,REGPTF,RELPTF,TAU,
     *    EGPTF,ELPTF,VDNPTF,PEPTF,PEPTFH,PEPTFL,PEPTFD,DPTF
C
      TYPE 73
      TYPE 74, (S(I),C1S(I),CS(I),F(I),X(I),Y(I),I=1,10)
      TYPE 75,R
C
C  CLOSE THE DATA FILE.
C
      CALL CLOSE(1)
      TYPE 65
      GO TO 10
 99   CALL CLOSE(2)
      STOP
C
C  INPUT FORMAT STATEMENTS
C
 15   FORMAT(1X,'RUN NUMBER?')
 30   FORMAT(I3,2X,I2,3X,I2,3X,7F5.1)
 31   FORMAT(7F5.1)
 32   FORMAT (2I1)
 35   FORMAT (2F5.1)
```

```
        C
        C  OUTPUT FORMAT STATEMENTS
        C
0189     65    FORMAT (1X,/)
0190     70    FORMAT(28X,'RUN NUMBER',I4//
             *     12X,'GAS FLOW RATE',14X,F9.4,2X,'CC/SEC'/
             *     12X,'SUPERFICIAL GAS VELOCITY',F12.4,'  CM/SEC'/
             *     12X,'SUP. GAS REYNOLDS NO.',F15.4/
             *     12X,'LIQUID FLOW RATE',11X,F9.4,2X,'CC/SEC'/
             *     12X,'SUPERFICIAL LIQUID VELOCITY',F9.4,'  CM/SEC'/
             *     12X,'SUP. LIQUID REYNOLDS NO.',F12.4/
             *     12X,'LIQUID SPACE TIME'/
             *     13X,'(NEGLECTING GAS PRESENCE)',F10.4,2X,'SEC'/
             *     39X,F9.4,2X,'SQUARES'///
             *     12X,'HOLDUPS NOT USING MEAN RESIDENCE TIME'/
             *     13X,'GAS',F32.4/
             *     13X,'LIQUID',F29.4/)
0191     71    FORMAT(24X,'BY ANALYSIS OF MOMENTS'//
             *     12X,'GAS VELOCITY',15X,F9.4,2X,'CM/SEC'/
             *     12X,'LIQUID VELOCITY',12X,F9.4,2X,'CM/SEC'/
             *     12X,'GAS REYNOLDS NO.',11X,F9.4/
             *     12X,'LIQUID REYNOLDS NO.',8X,F9.4/
             *     12X,'T BAR IN',19X,F9.4,2X,'SQUARES'/
             *     12X,'T BAR OUT',18X,F9.4,2X,'SQUARES'/
             *     12X,'MEAN LIQUID RESIDENCE TIME', F10.4,2X,'SQUARES'/
             *     12X,'GAS HOLDUP',17X,F9.4/
             *     12X,'LIQUID HOLDUP',14X,F9.4/
             *     12X,'VESSEL DISPERSION NO.',F15.4/
             *     12X,'PECLET NO.',17X,F9.4/
             *     12X,'PECLET NO. W.R.T. DP',7X,F9.4/
             *     12X,'AXIAL DISP. COEFF.',9X,F9.4///)
0192     72    FORMAT(18X,'BY EVALUATION OF TRANSFER FUNCTION'//
             *     12X,'GAS VELOCITY',15X,F9.4,2X,'CM/SEC'/
             *     12X,'LIQUID VELOCITY',12X,F9.4,2X,'CM/SEC'/
             *     12X,'GAS REYNOLDS NO.',11X,F9.4/
             *     12X,'LIQUID REYNOLDS NO.',8X,F9.4/
             *     12X,'MEAN LIQUID RESIDENCE TIME',F10.4,2X,'SQUARES'/
             *     12X,'GAS HOLDUP',17X,F9.4/
             *     12X,'LIQUID HOLDUP',14X,F9.4/
             *     12X,'VESSEL DISPERSION NO.',F15.4/
             *     12X,'PECLET NO.',17X,F9.4/
             *     12X,'95% CONFIDENCE INTERVAL'/
             *     15X,'HIGH VALUE',F23.4/
             *     15X,'LOW VALUE',F24.4/
             *     12X,'PECLET NO. W.R.T. DP',7X,F9.4/
             *     12X,'AXIAL DISP. COEFF.',9X,F9.4,2X,'CM SQ/SEC')
0193     73    FORMAT (//,6X,'S',9X,'C1(S)',7X,'C2(S)',8X,'F(S)',
             *     6X,'ABSCISSA',4X,'ORDINATE'/)
0194     74    FORMAT (6E12.4)
0195     75    FORMAT(1X/12X,'CORRELATION COEFFICIENT',4X,F9.4///)
        C
0196     14    FORMAT (' DID YOU REALLY USE ROTAMETER G',I1,'?',/)
0197     22    FORMAT (' DID YOU REALLY USE ROTAMETER L',I1,'?',/)
0198     95    FORMAT(Q,3A1)
0199           END
```

8.4 Tabulation of Operating Conditions, Holdups and Dispersion Coefficients

M_S (kg)	Injection Mode	U_L (cm/sec)	U_G (cm/sec)	ε_G	ε_L	ε^*	Peclet No.	Dispersion Coefficient** D (cm^2/sec)
0	gas	4.4	0	0	1.00 0.87	1.00 0.87		28.6
		8.8	0	0	1.00 0.38	1.00 0.38		65.7
		13.2	0	0	1.00 0.53	1.00 0.53		577
	liquid	4.4	0	0 0.06	1.00 0.89	1.00 0.95		36.3
		6.5	0	0 0.10	1.00 0.83	1.00 0.93		15.3
		9.1	0	0 -0.05	1.00 1.08	1.00 1.03		96.4
		13.2	0	0 0.18	1.00 0.67	1.00 0.85		-23.4
	tube/liquid	4.4	0	0 0.04	1.00 0.92	1.00 0.96		45.0
		6.5	0	0 0.18	1.00 0.68	1.00 0.86		-12.4
		9.1	0	0 0.08	1.00 0.85	1.00 0.93		69.1
		13.2	0	0 0.04	1.00 0.93	1.00 0.97		57.1

*For each U_L, the first row of holdups were calculated with the expanded bed height method and the second row with the tracer residence time method. $\varepsilon = \varepsilon_L + \varepsilon_G$.

**Both Peclet number and dispersion coefficient were from the analysis of moments method, $Pe = u_L d/D$.

M_s (kg)	Injection Mode	U_L (cm/sec)	U_G (cm/sec)	ε_G	ε_L	ε	Peclet No.	Dispersion Coefficient D(cm^2/sec)
2	gas	4.4	4.4	0.10	0.34	0.44		
				0.09	0.35	0.44	1.60×10^{-1}	36.0
		6.5	4.4	0.09	0.44	0.53		
				0.18	0.28	0.46	2.61×10^{-1}	40.7
		9.1	4.4	0.09	0.52	0.61		
				0.19	0.34	0.53	1.70×10^{-1}	72.0
		13.2	4.4	0.10	0.64	0.74		
				0.25	0.36	0.61	6.56×10^{-1}	25.4
	liquid	4.4	0	0	0.42	0.42		
				0	0.48	0.48	1.65×10^{-1}	25.4
			1.4	0.03	0.39	0.42		
				0.06	0.35	0.41	6.39×10^{-1}	9.18
			2.1	0.06	0.36	0.42		
				0.07	0.35	0.42	9.65×10^{-1}	6.08
			3.0	0.07	0.37	0.44		
				0.08	0.35	0.43	6.58×10^{-1}	8.76
			4.4	0.10	0.34	0.44		
				0.13	0.28	0.41	2.61×10^{-1}	28.0
			6.4	0.11	0.35	0.46		
				0.16	0.27	0.43	2.49×10^{-1}	30.0
			9.3	0.15	0.33	0.48		
				0.25	0.15	0.40	1.97×10^{-2}	676
		6.5	0	0	0.48	0.48		
				0	0.31	0.31	1.86×10^{-1}	51.0
			1.4	0.02	0.45	0.47		
				0.04	0.42	0.46	2.24×10^{-1}	31.3
			2.1	0.03	0.46	0.49		
				-0.01	0.54	0.53	9.85×10^{-2}	56.0
			3.0	0.06	0.46	0.52		
				0.12	0.35	0.47	4.76×10^{-1}	17.8
			4.4	0.09	0.44	0.53		
				0.15	0.33	0.48	9.59×10^{-2}	94.0

M_s (kg)	Injection Mode	U_L (cm/sec)	U_G (cm/sec)	ε_G	ε_L	ε	Peclet No.	Dispersion Coefficient D(cm²/sec)
2	liquid	6.5	6.4	0.11	0.42	0.53		
				0.24	0.18	0.42	7.78×10^{-2}	212
			9.3	0.14	0.41	0.55		
				0.15	0.39	0.54	4.09×10^{-2}	186
		9.1	0	0	0.57	0.57		
				0	0.03	0.03	2.22×10^{-4}	5.65×10^{5}
			1.4	0.02	0.56	0.58		
				0.09	0.43	0.52	2.72×10^{-1}	36.1
			2.1	0.06	0.53	0.59		
				0.17	0.32	0.49	1.08×10^{-1}	121
			3.0	0.06	0.53	0.59		
				0.20	0.28	0.48	2.08×10^{-1}	70.6
			4.4	0.09	0.52	0.61		
				0.18	0.34	0.52	5.24×10^{-2}	232
			6.4	0.10	0.49	0.59		
				0.27	0.18	0.45	8.92×10^{-2}	258
			9.3	0.14	0.48	0.62		
				0.30	0.19	0.49	1.86×10^{-2}	1200
		13.2	0	0	0.67	0.67		
				0	0.28	0.28	2.11×10^{-2}	1020
			1.4	0.02	0.67	0.69		
				0.14	0.44	0.58	2.90×10^{-1}	47.2
			2.1	0.05	0.65	0.70		
				0.29	0.22	0.51	6.34×10^{-1}	42.7
			3.0	0.07	0.64	0.71		
				0.25	0.33	0.58	5.80×10^{-1}	32.1
			4.4	0.10	0.64	0.74		
				0.26	0.35	0.61	9.06×10^{-2}	194
			6.4	0.13	0.62	0.75		
				0.41	0.11	0.52	1.45×10^{-3}	3.79×10^{4}
			9.3	0.16	0.59	0.75		
				0.24	0.44	0.68	1.26×10^{-2}	1090

M_s (kg)	Injection Mode	U_L (cm/sec)	U_G (cm/sec)	ε_G	ε_L	ε	Peclet No.	Dispersion Coefficient D(cm^2/sec)
2	tube/liquid	4.4	0	0	0.42	0.42		
				0	0.43	0.43	3.41	1.38
			2.1	0.07	0.36	0.43		
				0.05	0.38	0.43	$5.86x10^{-1}$	9.08
			4.4	0.10	0.34	0.44		
				0.08	0.37	0.45	$1.87x10^{-1}$	29.2
			9.3	0.15	0.33	0.48		
				0.12	0.38	0.50	$1.56x10^{-1}$	34.2
		6.5	0	0	0.48	0.48		
				0	0.46	0.46	-8.83	-0.737
			2.1	0.03	0.46	0.49		
				-0.06	0.62	0.56	$9.43x10^{-2}$	51.2
			4.4	0.09	0.44	0.53		
				0.01	0.59	0.60	$4.91x10^{-2}$	103
			9.3	0.14	0.41	0.55		
				0.13	0.42	0.55	$9.02x10^{-2}$	77.9
		9.1	0	0	0.57	0.57		
				0	0.48	0.48	$4.21x10^{-1}$	20.6
			2.1	0.06	0.53	0.59		
				0.18	0.31	0.49	$3.56x10^{-1}$	37.6
			4.4	0.09	0.51	0.60		
				0.33	0.09	0.42	$1.89x10^{-2}$	2500
			9.3	0.14	0.48	0.62		
				0.12	0.52	0.64	$6.87x10^{-2}$	117
		13.2	0	0	0.42	0.42		
				0	0.58	0.58	$2.143x10^{-1}$	49.0
			2.1	0.05	0.65	0.70		
				0.20	0.38	0.58	$2.72x10^{-1}$	59.3
			4.4	0.10	0.63	0.73		
				0.08	0.66	0.74	$5.82x10^{-2}$	157
			9.3	0.16	0.59	0.75		
				0.32	0.30	0.62	$4.47x10^{-1}$	45.8

M_s (kg)	Injection Mode	U_L (cm/sec)	U_G (cm/sec)	ε_G	ε_L	ε	Peclet No.	Disperion Coefficient D (cm^2/sec)
3	liquid	4.4	1.4	0.03	0.38	0.41		
				0.05	0.35	0.40	1.30	4.40
			2.1	0.06	0.36	0.42		
				0.07	0.33	0.40	-1.24	-4.96
			3.0	0.09	0.34	0.43		
				0.10	0.33	0.43	4.80×10^{-1}	12.8
			4.4	0.08	0.36	0.44		
				0.13	0.28	0.41	49.2	0.150
			6.4	0.11	0.34	0.45		
				0.17	0.23	0.40	-3.92×10^{-1}	-22.9
			9.3	0.13	0.35	0.48		
				0.14	0.33	0.47	3.76×10^{-1}	16.5
		6.5	1.4	0.02	0.46	0.48		
				0.08	0.36	0.44	3.10×10^{-1}	26.3
			2.1	0.05	0.45	0.50		
				0.17	0.22	0.39	1.81×10^{-1}	75.7
			3.0	0.08	0.41	0.49		
				0.27	0.08	0.35	6.11×10^{-1}	60.9
			4.4	0.08	0.43	0.51		
				0.15	0.31	0.46	1.88×10^{-1}	51.7
			6.4	0.12	0.39	0.51		
				0.21	0.23	0.44	1.01×10^{-1}	125
			9.3	0.14	0.38	0.52		
				0.14	0.38	0.52	9.22×10^{-2}	84.9
		9.1	1.4	0.03	0.55	0.58		
				0.15	0.33	0.48	1.12×10^{-1}	114
			2.1	0.06	0.52	0.58		
				0.16	0.34	0.50	1.13×10^{-1}	109
			3.0	0.07	0.50	0.57		
				0.12	0.41	0.53	1.25×10^{-1}	81.0
			4.4	0.09	0.51	0.60		
				0.20	0.31	0.51	1.22×10^{-1}	110

M_s (kg)	Injection Mode	U_L (cm/sec)	U_G (cm/sec)	ε_G	ε_L	ε	Peclet No.	Dispersion Coefficient D (cm^2/sec)
3	liquid	9.1	6.4	0.13	0.46	0.59		
				0.32	0.12	0.44	3.25×10^{-2}	1110
			9.3	0.15	0.40	0.55		
				0.38	-0.01	0.37	5.57×10^{-6}	-1.46×10^{8}
		11.7	1.4	0.03	0.62	0.65		
				0.13	0.44	0.57	1.94×10^{-1}	63.9
			2.1	0.06	0.60	0.66		
				0.20	0.35	0.55	4.69×10^{-1}	32.6
			3.0	0.10	0.49	0.59		
				0.14	0.41	0.55	5.35×10^{-2}	247
			4.4	0.11	0.58	0.69		
				0.22	0.37	0.59	8.67×10^{-2}	167
			6.4	0.12	0.57	0.69		
				0.26	0.33	0.59	1.64×10^{-1}	100
			9.3	0.15	0.55	0.70		
				0.23	0.41	0.64	4.91×10^{-2}	271

15. Stegel, G.J., and Y.T. Shah, "Backmixing and Liquid Holdup in a Gas-Liquid Cocurrent Upflow Packed Column," I&EC Proc. Des. Dev., 16, 37 (1977).

16. Wen, C.Y. and Y.H. Yu, "Mechanics of Fluidization," Chem. Eng. Prog. Symposium Series, 62, 100 (1966).

ORNL/MIT-250

INTERNAL DISTRIBUTION

1-2. J.M. Begovich
3. S.D. Clinton
4. H.D. Cochran
5. C.W. Hancher
6. J.R. Hightower
7. J.M. Holmes
8. Yvonne Lovely
9. J.P. Nichols
10. G.D. Robbins
11-12. J.S. Watson
13. C.D. Scott
14-15. Central Research Library
16. Document Reference Section
17-19. Laboratory Records
20. Laboratory Records, ORNL R.C.
21. ORNL Patent Office
22-36. MIT Practice School

EXTERNAL DISTRIBUTION

37. C.O. Reiser, Arizona State U., Tempe
38. J.E. Vivian, MIT, Cambridge, MA
39. Director, Research & Technical Support Div., ERDA, ORO
40-41. Technical Information Center